DIMENSIONALITY ASSESSMENT OF PROXIMITY-BASED DATA IN UNFOLDING MODEL APPLICATIONS

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DIMENSIONALITY ASSESSMENT OF PROXIMITY-BASED DATA IN UNFOLDING MODEL APPLICATIONS

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CHAPTER 1

INTRODUCTION

Dimensionality assessment is a chief concern within the field of psychological measurement. There is long-standing agreement that the ability to detect the dimensionality of a measure is a necessity (Hattie, 1958; Lord, 1980; Stout, 1987; Tate, 2003). At a minimum, any dimensionality assessment attempts to identify the number of underlying latent traits that the instrument (i.e., a test or questionnaire) is measuring. Moreover, within the latent variable modeling approach of item response theory (IRT), item level statistical assessment of dimensionality is imperative (Stout, 1996). Thus, it is not surprising that the more useful assessment methods not only identify the number of dimensions, but also describe what items measure each dimension.

The knowledge of a test's dimensionality affects model selection, the statistical analysis procedures used, and the interpretation of the results. When attempting to model latent traits, problems arise if the dimensionality of the model and data do not align (Ackerman, 1989; Chen & Thissen, 1997; Yen, 1985). Because of the abundance of unidimensional measurement models, when analyzing the dimensionality of a test, often the initial task is to determine if the test is unidimensional. Misdiagnosing the unidimensionality of a set of data can result in excluding extra factors that are present in the data and the misapplication of measurement and statistical programs (e.g., BILOG, PARSCALE, MULTILOG, LOGIST), which in turn, can lead to inaccurate parameter estimates. Beyond discerning if the test is unidimensional, there is also a need to describe the correct multidimensional structure of the test (Green, 1983). The ability to

explain the multidimensional structure of data will facilitate a better understanding of the test results, the test itself and lead to better specified multidimensional IRT models (MIRT).

For example, suppose a test is administered to assess the mathematical proficiency of students. The test items are word problems, suggesting the test measures mathematical ability and reading ability (i.e., the test is two-dimensional). If the data were modeled with a unidimensional model (i.e., a model that assumes the test only measures mathematical ability), the resulting estimates will not be accurate and the interpretation of the results would be incorrect. The resulting biased estimates are due in part to the fact that assuming data are unidimensional leads to the exclusion of possible confounding or extra factors that lead to item covariance that is not explained by the model, and thus, not part of the likelihood function. Additionally, if the data were not unidimensional, then certain measurement programs that assume unidimensionality, such as the programs mentioned above, would be inappropriate. In this case, a dimensionality assessment tool with good performance would aid in avoiding these problems.

The current review focuses on (a) presenting the idea of unfolding data as they differ from cumulative data and (b) reviewing the current collection of dimensionality assessment tools with attention paid to their appropriateness in the context of proximity-based responses. Most dimensionality assessment tools have been designed for responses from a dominance-based response process, which are analyzed with a cumulative model. Unfortunately, these tools are typically inappropriate for proximity-based responses like those analyzed with an unfolding model. Therefore, there is a need for a robust dimensionality assessment tool for unfolding data. The proposed research will present and test a simple dimensionality assessment tool that satisfies this need.

Unfolding Models

Traditional IRT models work under the assumption that the latent trait level of the respondent is monotonically related to the probability of endorsing an item. These models can be described intuitively as "more is better models," and in psychometrics they are often referred to as monotone or cumulative models. These types of models most appropriately describe item response data that follow the assumption mentioned above (i.e., result from a dominance-based response process). These data can be routinely found in the measurement contexts involving academic proficiency, personality traits, and clinical diagnoses. As described above, the educational testing framework assumes that the probability of providing the correct answer on a mathematics exam is monotonically related to latent math proficiency. As the estimated trait level of an individual increases, so does the probability of getting a math test item correct. Such models would yield monotonically increasing item characteristic curves (ICC) as seen in *Figure 1.1*.

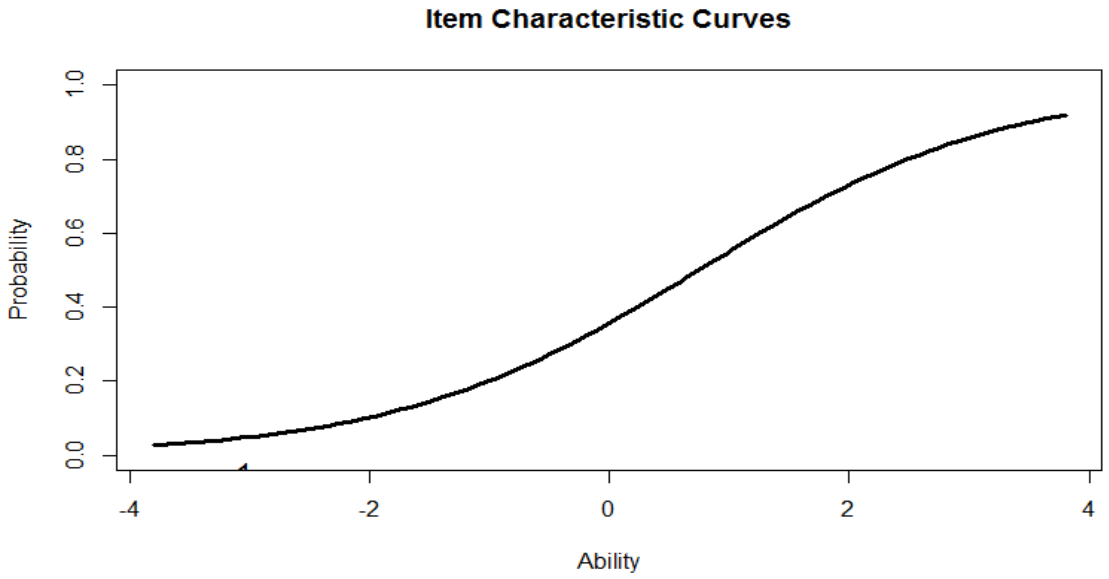


Figure 1.1. Item characteristic curve for a hypothetical item that follows a cumulative model.

However, not all item response data conform to a cumulative model. There are other areas in psychology where item responses generally follow from a proximity-based process (a.k.a. an ideal point response process). These areas include measurement of attitudes, preferences, and certain developmental changes that occur in distinct stages (Noel, 1999; Stark et al., 2006; Tay et al., 2009). Thurstone's (1928) work is a classical illustration that implicitly presumes that responses to attitude questionnaires specifically follow from an ideal point process. Following Thurstone's seminal work, there have been various confirmations throughout the years that responses to Thurstone and Likert style attitude questionnaires do indeed follow this process (Andrich, 1996; Roberts, Laughlin & Wedell, 1999; Van Schurr & Kiers, 1994). The measurement of the aforementioned psychological constructs is a frequently researched area within psychology. As a result, there have been different models proposed for item response data that follow from an ideal point process (Andrich, 1988, 1996; Luo, 2001; Roberts et al., 2000). Coombs (1964) referred to models for ideal point responses as "unfolding models."

The notion behind ideal point processes is that a person will endorse an item to the degree that the person and the item are located near each other on the underlying latent trait continuum or latent space. In other words, the endorsement probability increases as the distance between an item location and a person's ideal point approaches zero, and the probability decreases as this distance increases in any direction. The ICC of a unidimensional unfolding item would have a peak (fold) at the point on the latent trait continuum where the person and item location are equal. It is at this point that an ICC exhibits it maximum value. The ICCs of various unfolding scale items located along the latent trait continuum are presented in *Figure 1.2*. The items are separated into different plots based on their relative location on the latent trait continuum. By examining the neutral (moderate) items, the fold of the ICC can be seen clearly. Although the

extreme items' ICCs look like that of a cumulative item, the fold would be visible theoretically for these items if the horizontal axis could encompass more of the latent trait continuum. In regard to *Figure 1*.2, it is important to note that if the horizontal axis were rescaled by subtracting the item's location from the person's location, a bell shaped, unfolding ICC occurs for every item.

Not unlike cumulative data, a set of unfolding data has a particular dimensional structure. Accordingly, dimensionality assessment is equally important for a set of unfolding data as it is for a set of cumulative data. The next section discusses the methods by which dimensionality is currently assessed for both types of data.

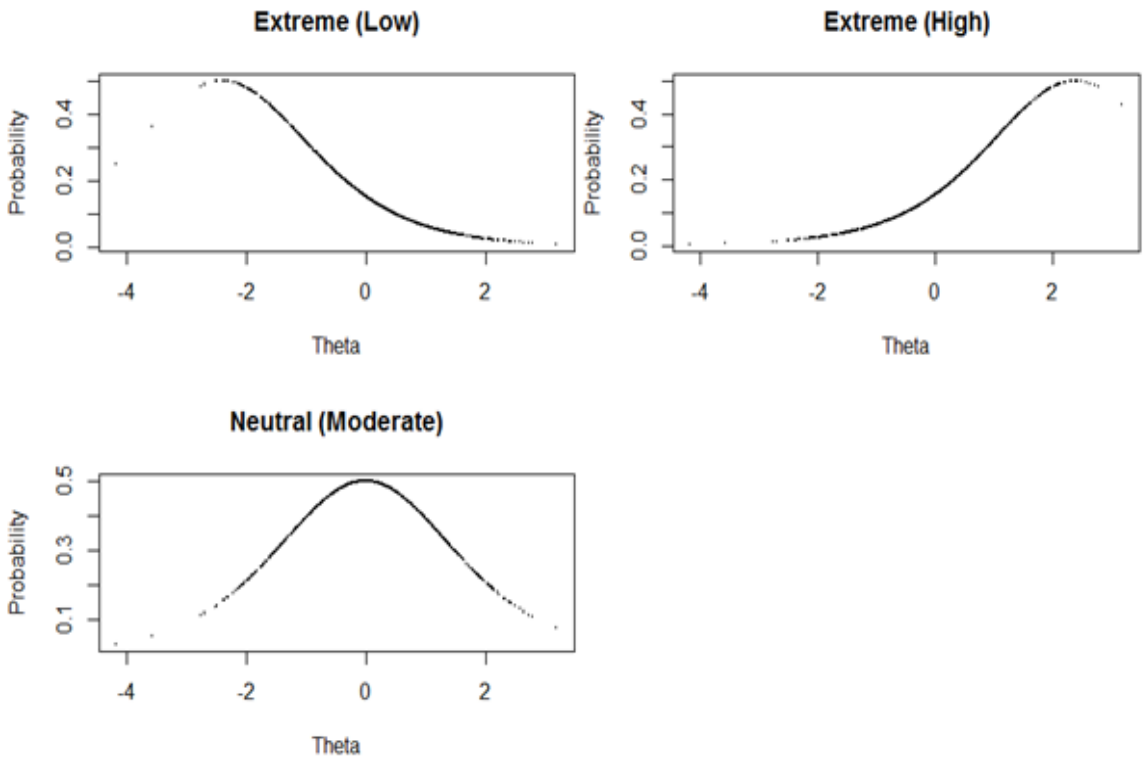


Figure 1.2. Item characteristic curves for three hypothetical unfolding items

Unidimensionality Assessment Tools

There are a plethora of unidimensionality assessment tools that have been developed for cumulative data. These methods attempt to assess the hypothesis of unidimensionality. That is, these methods attempt to diagnose if the test is in fact only measuring one dimension. Some of the simplest and well-known ways to assess the dimensionality of data is by principal components analysis (PCA) and factor analysis (FA).

Both of the methods just mentioned are linear models that reduce data into fewer components or factors, respectively. The first step either method is an eigenvalue/eigenvector decomposition of a square, symmetric matrix. One major difference between PCA and FA lies in the type of matrix that is decomposed. In PCA, the matrix that is traditionally decomposed is a correlation matrix, whereas the decomposed matrix in FA is a reduced correlation matrix (i.e., it contains communality estimates along the main diagonal instead of ones). Because of this difference, the complete set of principal components will account for the total amount of variance present in the data, while the full set of factors will account for the common variance in the data. However, both sets (principal components and factors) will be correspond to the calculated eigenvalues from their respective matrices, and will be in descending order. The first principal component (or factor) will have a corresponding eigenvector that indicates a direction in space that accounts for the most variance (or common variance) in the data, the second will account for the next largest amount of variance (or common variance), and so on. These principal components and factors show the underlying structure of the data. However, unlike FA, PCA is a mathematical identity, which, by definition, orients the data space such that each dimension corresponds to orthogonal directions that account for the largest amount of variation

in the data. Therefore, it is not possible to rotate the PCA solution and maintain this identity, whereas rotation of the solution is commonly seen in FA.

After this first step, one must decide on the number of dominant dimensions to retain by examining eigenvalues that correspond to each principal component or factor to reduce the data. The number of dominant eigenvalues that underlie a set of data indicates the dominant dimensions within the data. The idea is to choose the smallest number of dominant dimensions that still account for a significant amount of (total or common) variance in the data. Interestingly, determination of the number of dominant dimensions has typically been based on a PCA solution regardless of whether a FA solution is the ultimate goal.

When choosing the dominant eigenvalues that underlie a set of data, one must use some decision criteria to justify the choice. Consequently, there are several proposed decision criteria used in PCA. For example, one could use Cattell's scree test (Cattell, 1996) or Kaiser's rule (eigenvalues greater than one; Kaiser, 1960). One of the better performing methods is a bootstrapped version of Horn's parallel analysis procedure (Horn, 1965; Lambert et al., 1990). In this procedure, data are randomly sampled from the existing data set, with replacement. Thus, the data are sampled from the empirical distribution of the observed data. The eigenvalues of the correlation matrix for random data are compared to those derived from the original real data. The eigenvalues from the real data that are greater than those found from random data are retained. If only one dominant eigenvalue is retained from any of these decision methods, the data are assumed to be unidimensional. Any larger number of dominant factors would indicate a multidimensional data set.

Although FA and PCA are appropriate in many analytic situations, the two procedures do have their limitations. The issues with factor analysis and principal components analysis on

dichotomous IRT response data have been well documented (Bernstein & Teng, 1989; McDonald, 1981; Reise, 1999; Waller et al., 1996). One such problem is the existence of what have been called "difficulty factors". These difficulty factors occur because binary IRT data often violate the primary assumption of linearity in factor analysis—the assumption that there exists a linear relationship between observed variables and the underlying latent trait(s). When assumptions of linearity are violated, spurious dominant factors, or difficulty factors, can appear because items with similar difficulty tend to form additional factors distinct from the true dominant underlying dimension(s), thus resulting in overestimation of the true dimensionality of the data.

This problem has led to the development of dimensionality assessment tools that do not rely on principal component or factor analytic procedures. Many of the widely accepted unidimensionality assessment tools for cumulative data are formed on the basis of conditional covariance. The conditional covariance based procedures described below are also nonparametric, meaning that there is no assumption of a mathematical (parametric) form other than the assumption of monotonicity between the item response and the latent trait(s).

The use of conditional covariance based methods is related to the assumption of local independence between test items. There are two principles of local independence that are used: strong and weak local independence. The assumption of strong local independence is that any and all subsets of items on a test are locally independent, after conditioning on all of the underlying latent traits. Hence, for a unidimensional test, a single latent trait is assumed to be providing all of the information behind an individual's performance if local independence is achieved by conditioning on only one latent trait. If the test is not unidimensional, then the local independence assumption will be violated when conditioning on exactly one latent trait.

Several authors have posited that test data are rarely unidimensional, especially when unidimensionality is assessed using the strict strong local independence assumption above (Humphreys, 1985; Yen, 1985). The methods presented below rely on less strict definition of local independence; the second principle of local independence mentioned above termed weak local independence or pairwise local independence. Test data exhibit weak local independence if for every unique item pair, and for every θ, the conditional covariance is equal to zero (Stout, 1987). It has been noted that in real test data, if weak local independence holds, then strong local independence holds approximately (Stout, 1996). This definition of weak local independence is closely related to the concept of essential independence—a more relaxed definition of independence. Essential independence does not require that every item pair have a conditional covariance of zero; only that, as the number of items goes to infinity, the average conditional covariance between item pairs is small (Nandakumar, 1991).

As previously indicated, these principles of local item independence are foundational for the conditional covariance based methods of dimensionality assessment used for cumulative data. In the following paragraphs, I will explain three of these dimensionality assessment tools: HCA/CCPROX, DIMTEST, and DETECT.

The first step of the HCA/CCPROX procedure is a hierarchical cluster analysis. The object in any hierarchical analysis is to group objects, or in this case items, that are "close" to one another by repeatedly calculating some distance measure between objects and between clusters once objects begin to be clustered together. In the initial clustering of items in HCA/CCPROX, every item is in its own cluster. In each subsequent step of the cluster analysis, clusters of items are grouped together based on a conditional covariance proximity measure

(CCPROX), using the unweighted pair-group method of averages to join clusters (Roussos, 1995b). The proximity measure used in HCA/CCPROX is defined as:

$$\rho_{ccov} = \frac{-1}{\sum n_k} \sum_{k=0}^{N-2} n_k \widehat{cov}\left(U_i, U_l \middle| S_{i,l} = k\right) + constant, \qquad (1)$$

where $S_{i,l}$ is the examinee's number correct score on the remaining n-2 items after excluding items i and l, N_k is the number of examinees with $S_{i,l}=k$, and $\widehat{cov}$ is the standard maximum likelihood estimation of the covariance.

The HCA/CCPROX procedure continues to join clusters until all items are in one cluster. The goal is to reach a step in the cluster analysis where items clustered together are believed to measure the same latent trait and each trait is indexed by the items in a single cluster. Thus, the intended result is that the number of clusters at that step will equal the dimensionality of the test. The problem with this technique is identifying the point in the procedure when the number of clusters that remain indicates the correct dimensionality of the test.

DIMTEST is a nonparametric procedure that assumes unidimensionality and then checks for approximate local independence between clusters of test items (Stout, 1987; Nandakumar & Stout, 1993). The first step of the (original) DIMTEST procedure is to divide the test into three clusters of items. The three item clusters used in DIMTEST are called the assessment subtest (AT1), assessment subtest 2 (AT2), and the partitioning test (PT). The set of items in AT1are selected to be homogeneous and as distinct as possible from the remaining items. Originally, the selection of items for AT1 was done through expert opinion or exploratory factor analysis. The AT2 consists of items that are not on AT1, but have a similar difficulty distribution as AT1. Additionally, items included in AT2 contain the same dimensional structure as the PT. The PT contains the remaining items, and should be more or less monotonically related to AT2.

Examinees' scores on PT are used to assign individuals to homogeneous subgroups. In other words, the examinees are segregated based on their scores on PT.

After constructing the three tests, the final step is to examine the item covariance of the two assessment tests after conditioning on the examinees' scores from each of the subgroups of the PT. If the test is unidimensional, then after conditioning on a particular subgroup of the PT, there should be approximately zero item covariance in AT1. However, if the AT1 represents a different dimension than the PT, then nonzero item covariance will be present in the AT1. Thus, if unidimensionality holds, the examinees with the same PT score will produce approximate weak local independence on the AT1 (Stout, 1996). Note that the approximation is confounded somewhat due to difficulty factors along with unreliability in PT scores. Therefore, the AT2 item covariance is also evaluated. Any item covariance that is displayed in the AT2 is due to difficulty factors and a lack of reliability of the PT scores. The estimate of item covariance in AT2 is used to correct the biased item covariance observed in AT1 due to these factors.

Recent developments in the DIMTEST procedure have led to better selection of the AT1 and PT, as well as the exclusion of the AT2 entirely (Froelich & Habing, 2002, Stout et al., 2001). In the more recent DIMTEST, potential item clusters for the AT1 are selected using the HCA/CCPROX procedure. The best candidate from these selections is found by calculating the maximum DETECT index (to be discussed next). Moreover, there is no need for the AT2 because a new bias correction method is used based on resampling techniques. This increases the number of items remaining for the PT.

The third method, DETECT, is a statistical estimation procedure that uses the DETECT index (Kim, 1994; Zhang & Stout, 1999). The DETECT index is a measure of the amount of multidimensionality present in a particular partition of the test. In this procedure, the goal is to

find a collection of mutually exclusive and exhaustive item subsets (a.k.a. a partition) that maximize the DETECT index, producing the partition with the most dimensionally distinct clusters. While the unidimensionality of the data can be assessed with the DETECT measure, the multidimensionality of the data cannot always be described because the number of dimensionally distinct clusters produced by the DETECT procedure is not always equal the correct number of dimensions of the data (Stout, 2002; Zhang & Stout, 1999). This is because when classifying items into dimensionally homogenous clusters in the DETECT procedure, all items being dominated by a particular ability can be considered to measure the same ability, and are thus grouped together even if they measure, to a lesser degree, other constructs as well (Zhang & Stout, 1999). For example, suppose performing the DETECT procedure resulted in two dimensionally distinct and individually homogenous clusters, but the items within each cluster also measure a third dimension, to a lesser degree. In this case, the DETECT procedure might suggest a two cluster partition, when in fact there are three underlying dimensions. Therefore, if the test items do not adhere to simple structure, the DETECT procedure cannot be reliably used to identify the number of true dimensions underlying multidimensional data (Stout, 2002).

All of the measures reviewed thus far attempt test or describe the dimensional structure of cumulative data. In the next section, I will discuss why factor analytic procedures produce erroneous results in addition to difficulty factors for unfolding data. Additionally, I will illustrate why these conditional covariance based procedures are not appropriate for unfolding data.

Problems when Applying Traditional Dimensionality Assessment Methods to Unfolding Data

To begin, in addition to the difficulty factors seen from FA or PCA with dichotomous IRT data, unfolding data have an extra factor problem. The works of Davison (1977), van Schurr &Kiers (1994) and Maraun & Rossi (2001) discuss the presence of an extra factor when examining unidimensional unfolding data with factor analytic methods. This is due to the fact that, in the unfolding model, the data represent distances between items and persons, which can lead to serious interpretation issues of any factor analytic results (Tay & Drasgow, 2012; Van Schurr & Kiers, 1994). Although these observations provide some knowledge of the problem, there is no investigation into how this "extra factor problem" is manifested in contexts with multidimensional ideal point data in which either simple of complex item structure may be present.

When considering all of the conditional covariance based techniques in the context of unfolding data, the most problematic feature of these methods is that they all use the examinee's number correct score (NCS) or the number of remaining items score (rest score) across test or subtest items as a proxy for the latent trait. Thus, the above methods assume a monotonically increasing relationship between the NCS and the probability of correct response or endorsement. To reiterate, this suggests that the more items an individual gets correct, no matter which items, then the higher the latent trait should be. Ideal point data violate this assumption. When data follow from an ideal point process, the number correct might be the same for two individuals, but that does not necessarily indicate that they both have the same trait level because item responses are nonlinearly related to the latent trait, and thus, sums of those responses would generally be nonlinearly related to the trait as well.

To illustrate, consider two respondents who each endorsed three items out of ten total items on an attitude questionnaire. Furthermore, suppose one respondent endorsed items that are all located on the negative end of the attitude continuum, and the other endorsed items located on the positive end of the attitude continuum. As previously stated, attitude responses are commonly thought to follow from an ideal point response process; thus, subjects will endorse items that are located close to their location on the attitude continuum. Therefore, it is necessary to know the location of the items endorsed by each respondent to best estimate their latent trait level. The total score alone does not provide this information.

In this example, if latent trait estimation were based on total score alone, our two respondents would have the same estimated latent trait level. However, after acquiring the location of the items endorsed, it is clear they do not share the same latent trait level. Thus, the NCS is not monotonically related to the latent trait under general circumstances. Therefore, it is inappropriate to use these conditional covariance based dimensionality assessment tools (which condition on NCS or the rest score) when the data follow from an ideal point process, despite the overall good performance of these measures with data arising from a dominance-based response process. (Van Abswoude, Van der Ark & Sijitsma, 2004).

As identified previously, there is a large body of substantive areas in which the data follow from an ideal point process. Within these contexts, dimensionality assessment still remains a primary concern. At this point, there exist only a few suggestions to aid in the dimensionality assessment of unfolding data, including the factor analytic observations mentioned above. Moreover, the only proposed dimensionality assessment tool for unfolding IRT response data that uses a statistical test of a hypothesis is an adaptation of Yen's Q_3 for unfolding data (Habing, Finch & Roberts, 2005). While this adaptation of Yen's Q_3 for

unfolding data has some promise, more research is needed to explore its application to a wider variety of situations. Additionally, this measure is complex and exhibits biases under certain conditions. There is a need for a dimensionality assessment tool that does not sacrifice simplicity and demonstrates good performance under diverse conditions for unfolding IRT response data.

A dimensionality assessment tool has been found for unfolding data in the multidimensional scaling domain. Coombs and Kao (1960) postulated that if a PCA were performed on distances between stimuli, then there would be *r+1* dominant dimensions where *r* is the number of true dimensions. Ross and Cliff (1964) went on to mathematically prove this hypothesis: if a PCA is performed on squared distances between stimuli, there will be *r+1* dominant components where *r* is the true dimensionality of the scale. Both findings were the foundation for the studies of the "extra factor problem" mentioned above. The type of data used by Coombs & Kao and Ross & Cliff were preference rank orders for stimuli. While this finding is pivotal for multidimensional unfolding scales, the proof of this principle is not directly transferable to polytomous unfolding IRT response data because polytomous unfolding IRT response data are not equivalent to preference rank orders of stimuli. These two types of data are qualitatively different.

More specifically, with preference rank orders the subjects are comparing the stimuli to each other and to themselves; constructing a scale that orders the stimuli. On the other hand, with polytomous unfolding IRT response data subjects are comparing the stimulus (item) to themselves (i.e., their location on the latent trait continuum) one item at a time. Additionally, ranking data are inherently polytomous and do not allow for ties. In contrast, IRT responses are discrete and ties are generally unavoidable. In short, this dimensionality assessment method first

suggested by Coombs and Kao (1960) has not been seriously explored for unfolding IRT response data.

This research study will attempt to show that a PCA on item responses resulting from an ideal point response process will generally result in $r+1$ dominant dimensions under a variety of conditions. In particular, it will assess whether PCA can be used to determine the dimensionality of item responses that are consistent with the Multidimensional Generalized Graded Unfolding Model (MGGUM; Roberts & Shim, 2010). If this finding holds true, it will provide researchers with a simple dimensionality assessment tool for unfolding IRT data. Additionally, this simple dimensionality assessment tool will enable psychometricians to apply the MGGUM in common psychological measurement contexts, such as attitude and personality assessment, and consequently increase measurement validity.

CHAPTER 2

METHOD

Simulation Design

A simulation study was performed to determine the viability of PCA combined with the *r+1* heuristic as a dimensionality assessment tool for unfolding IRT responses. This simulation study investigated the effects of six factors on the identification of the true dimensionality of tests using this simple dimensionality assessment tool. The six factors were the (a) sample size, (b) test length, (c) number of response categories, (d) true dimensionality of the test, (e) type of multidimensional structure, and (f) correlation between the true underlying dimensions of the test in the instances where the simulated test is multidimensional. Sample size was comprised of four levels: 500, 1,000, 1,500 and 2,000 examinees. The number of test items, response categories, and correlations between dimensions each had three levels. The number of test items was comprised of 20, 40, or 60 items. The response categories for each item varied among 2, 4, or 6 response categories. The correlation among dimensions varied between 0, .3, and .7. The dimensionality of the test also had three levels: unidimensional, two-dimensional and three-dimensional. Finally, the multidimensional structure of the two and three dimensional tests had two levels: simple and complex (i.e., between item and within item multidimensionality). The sample size, number of items, and response category factors were fully crossed for each level of the dimensionality factor. For two and three dimensional data sets, the multidimensional structure and the correlation between dimensions factors were fully crossed in addition to the previously mentioned three factors. The simulation contained 1,000 replications in each of the resulting 468 cells.

Data Generation

The data was generated with the Multidimensional Generalized Graded Unfolding Model (MGGUM) (Roberts & Shim, 2010). The MGGUM is defined as follows:

$$P\left[Z_i = z \middle| \underline{\theta}_j\right] =$$

$$\frac{\exp\left[\left(z\sqrt{\sum_{d=1}^{D}\alpha_{id}^2(\theta_{jd}-\delta_{id})^2}\right)+\sum_{k=0}^{z}\psi_{ik}\right]+\exp\left[\left((M-z)\sqrt{\sum_{d=1}^{D}\alpha_{id}^2(\theta_{jd}-\delta_{id})^2}\right)+\sum_{k=0}^{z}\psi_{ik}\right]}{\sum_{w=0}^{C}\left(\exp\left[\left(w\sqrt{\sum_{d=1}^{D}\alpha_{id}^2(\theta_{jd}-\delta_{id})^2}\right)+\sum_{k=0}^{w}\psi_{ik}\right]+\exp\left[\left((M-w)\sqrt{\sum_{d=1}^{D}\alpha_{id}^2(\theta_{jd}-\delta_{id})^2}\right)+\sum_{k=0}^{w}\psi_{ik}\right]\right)} \quad (2)$$

and

$$\psi_{ik} = \sum_{d=1}^{D}\alpha_{id}\tau_{ik} \quad (3)$$

where θ_{jd} is the location of the *j*th individual on the *d*th dimension, δ_{id} is the location of the *i*th item on the *d*th dimension, αid is the discrimination parameter of the *i*th item on the *d*th dimension, and τ_{ik} is the *k*th subjective category threshold for the *i*th item. This latter parameter is constant across dimensions due to model identification requirements.

The true item parameters were randomly sampled with replacement from a list of parameters for the unidimensional Generalized Graded Unfolding Model (GGUM) found in Thompson (2014). The true individual locations on each dimension were sampled from a multivariate normal distribution with zero means and unit (1) variances. For the multidimensional cases, the tests were simulated such that they have different multidimensional structure (i.e., simple and complex). Simple structure occurs in a test if its items can be partitioned into item clusters that are each relatively dimensionally homogeneous and that are

dimensionally distinct; in this case, the number of clusters is equal to the number true dimensions. For the two-dimensional tests, simple structure was induced by having odd-numbered items measuring one dimension and the even-numbered items measuring the second dimension; thus, creating two dimensionally homogenous clusters that are also dimensionally distinct. The three-dimensional data had simple structure that is produced by a similar method (i.e., the test was divided into three subsets, with each subset containing items that measure a different dimension). In the case of simple structure, each item only had one nonzero item location and corresponding discrimination parameter.

To construct the tests to have complex structure, the location of each item on every dimension was nonzero. The item parameters were sampled, with replacement, from the same list of parameters mentioned above. However, in an effort to equate the amount of maximum discrimination across simple and complex structure conditions, the sampled α_{id} parameters were transformed by $\frac{1}{\sqrt{D}}$, where D is true number of dimensions underlying the generated dataset. This ensured that the maximum discrimination for complex structure items (i.e., the sum of α-id2; Reckase, 2009) approximates that for simple structure items (which have only a single nonzero discrimination parameter). Additionally, the τ_{ik} parameters were averaged across each sampled item because this parameter is a dimensionless quantity.

By using the MGGUM, a vector of item response probabilities was obtained for each individual in a given replication within the experimental design. From these item response probabilities, explicit item responses were generated that correspond to one of the objective response categories. These item responses were analyzed to determine the performance of the proposed dimensionality assessment tool. All item responses were generated using the SAS statistical package.

Assessing Dimensionality

A principal components analysis (PCA) was performed on the generated item responses. The number of dominant eigenvalues estimated by the PCA of each replicated data set was computed using a bootstrap parallel analysis procedure that calculates eigenvalues for responses that were randomly selected from the observed data with replacement (Lambert et al., 1990). Generation of random data and the calculation of corresponding eigenvalues were replicated 1,000 times for a given MGGUM data set. Instead of the traditional mean eigenvalue of the random data, the 75th, 95th, and 99th quantiles of the resulting eigenvalue distributions were used as alternative cutoff values to determine whether the corresponding observed eigenvalue should be retained (Buja & Eyuboglu, 1992). The number of dominant eigenvalues determined by this procedure constituted the $r+1$ dominant dimensions suggested by Coombs and Kao (1960), and thus, the estimated number of dimensions would be equal to r. The dependent variable of interest was the proportion of MGGUM data sets within a given cell of the design that are classified as one dimensional, two dimensional, three dimensional, four dimensional, etc. according to this decision criterion. These proportions were then compared to the true dimensionality of each MGGUM data set. A descriptive analysis of the results was performed.

Expected Results

Ad hoc tests have indicated that PCA of item responses generated with the MGGUM can provide acceptable results. That is, the PCA will result in $r+1$ dominant dimensions a high proportion of the time. It is expected that the procedure will perform better for tests containing more items and for larger sample sizes. Furthermore, the procedure is expected to perform worse

for binary items (i.e., items with two response categories) relative to cases with four or more response categories. Additionally, correlation among multiple dimensions is expected to make dimensionality assessment with the proposed method more difficult.

Summary

Dimensionality assessment of measurement instruments is no easy task. Within the field of IRT, much attention has been given to the issue of dimensionality assessment for item responses that are consistent with cumulative models. This has resulted in multiple, well-performing dimensionality assessment procedures for these models. However, there has been very little attention paid to dimensionality assessment for unfolding data. The development of a simple, yet accurate, dimensionality assessment tool for ideal-point responses is crucial to proper utilization of multidimensional unfolding IRT models like the MGGUM, and will increase the practicality of the MGGUM in common psychological research areas.

CHAPTER 3

RESULTS

The results of the simulation study are presented in Appendices A-D, where each appendix portrays the results according to each cutoff value for the random eigenvalue distribution used in the bootstrap parallel analysis. When examining the results across the appendices, it can be seen that as the cutoff value decreases from the 99^{th}, 95^{th}, and 75^{th} quantile to the mean, the proportion of replications that are overestimated (found to have greater than *r+1* dominant dimensions) increases. However, this difference is negligible. Consider Tables A.4, B.4, C.4 and D.4 from each appendix, which portray the number of dominant dimensions suggested by bootstrap parallel analysis of 2-dimensional, 20 item, MGGUM datasets with complex item structure and varying number of response categories and axis correlations. As seen in the first three rows of the first column in each table, the classification of the replications within this cell as three dimensional only varies from 988 to 998 as the cutoff value increases from the mean to the 99^{th} percentile, with the rest of the replications being classified as four dimensional. Across all conditions, this difference across the cutoff values is, on average, only approximately 50 replications per cell. Therefore, without loss of generality, the results presented below reflect outcomes using the 95^{th} percentile (i.e., tables in Appendix C) to avoid any, albeit minor, overestimation of dimensionality.

One Dimensional Data

For data that were generated to be truly one dimensional, the simulation study factors of sample size, test length and number of response categories were fully crossed. All of the

replications of each of these resulting cells were classified as having two dominant dimensions, regardless of the level of any of the aforementioned factors (i.e., 100% of the replications in each cell indicated *r+1* dominant dimensions, where *r* is the true dimensionality of the data). Additionally, it should be noted that this result held no matter the cutoff value of the random eigenvalue distribution from the bootstrap parallel analysis (See Tables A.1-A.3, B.1-B.3, C.1-C.3, and D.1-D.3).

Two Dimensional Data

Complex Structure

The results for all cells containing two dimensional datasets with complex structure are in Tables C.4-C.6, one table for each level of the test length factor. Tables C.4 and C.5 represent all data with test length of 20 and 40 items, and these results show that the percentage of replications within each cell classified as three dimensional (*r+1*) is over 85%; and excluding three cells, this percentage increases to over 90%. However, the bootstrap parallel analysis results in Table C.6 (test length of 60 items) show that only when sample size is equal to 500 does the percentage of replications classified as *r+1* mirror that seen in Tables C.4 and C.5. As sample size is increased, this percentage in Table C.6 being classified as *r+1* decreases from 80%, seen previously, to as low as approximately 35%, where the majority of replications are now classified as four dimensional. Also, there is a quadratic relationship between the number of response categories and the percentage of replications classified as *r+1* (3). That is, four response categories results in the largest number of replications classified as *r+1*, and this percentage drops for both two and six response categories (See Table C.6). These results are in the opposite direction of the anticipated results, specifically that this procedure will perform better for larger sample sizes and longer tests. Furthermore, it is not clear from these tabulations

whether it is the *r+1* rule, the identification of dominant dimensions using the bootstrap parallel analysis technique, or both that falter in these conditions.

To investigate these unexpected results further, a randomly selected sample of 100 datasets from the cell with the highest percentage of replications classified as four dimensional from Table C.6 were analyzed using Cattell's scree test (Cattell, 1966). This sample of scree plots was evaluated by two independent undergraduate student raters after a suitable amount of training. Of this sample of scree plots, 89% were agreed upon by both raters to have three

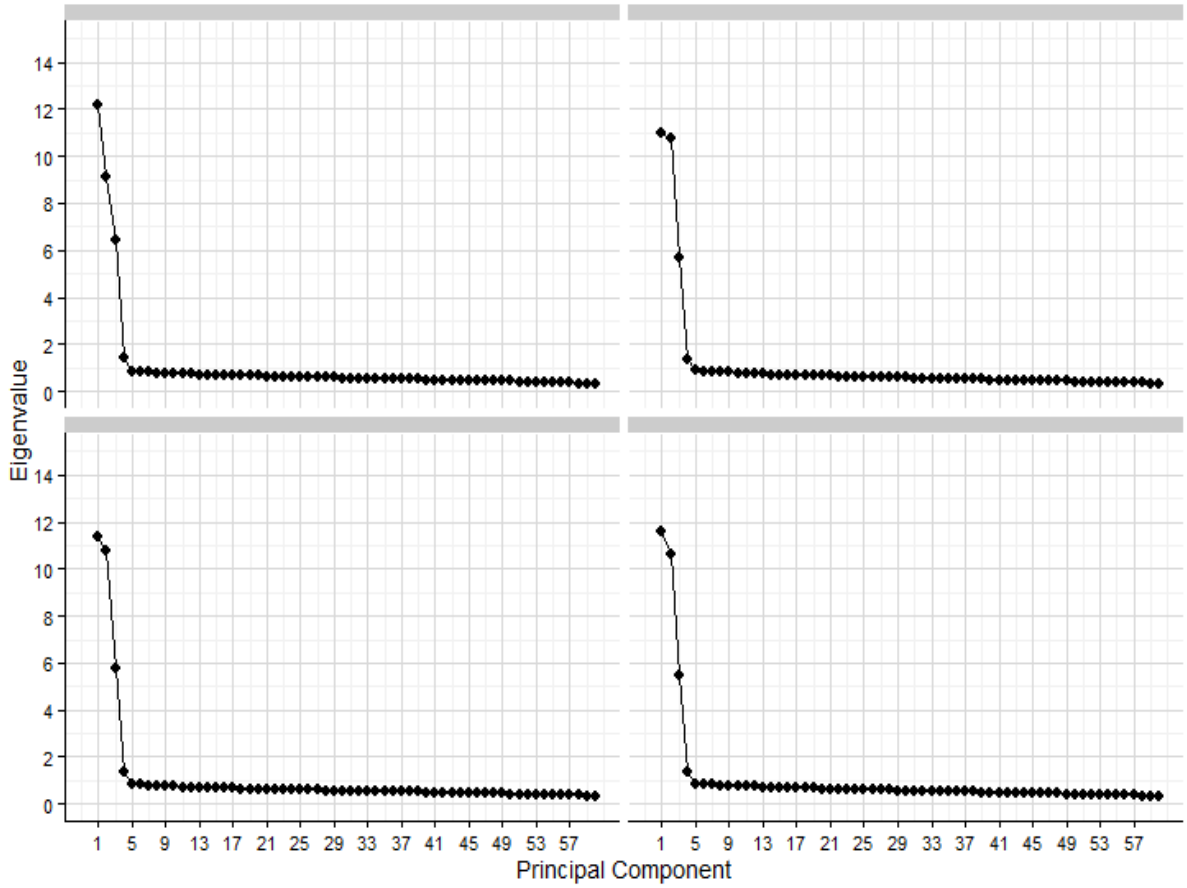


Figure 3.1. Scree plots for four randomly selected replications. Data are from a simulation study cell with 2 true dimensions, 60 items, complex structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

dominant dimensions, while one of the raters classified 100% of the sample as three dimensional (*r+1*). Figure 3.1 displays four of the scree plots from this sample. As Figure 3.1 shows, the scree plots clearly illustrate that these replications have *r+1* (3) dominant dimensions.

Based this visual exploration of a sample of replications within this cell, the PCA is indicating *r+1* dominant dimensions, and it is the identification of these dominant dimensions by the bootstrap parallel analysis procedure that is problematic. The poor performance of bootstrapped parallel analysis for large tests (i.e., overestimation of the number of dominant dimensions) has been documented previously (Roberts and Samuelson, 2002). It is expected that if all of the replications within Table C.6 were analyzed using Cattell's scree test, the results would be similar to those in the Tables C.4 and C.5, if not better, and hence, the *r+1* rule still appears to work well.

Simple Structure

The results for two dimensional simple structure datasets can be found in Tables C.7-C.9. For the datasets with 20 items (Table C.7), the results showed that data from over 70% of the replications in all of these cells were identified as four (2*r*) dimensional, and the actual percentage of such replications increased with sample size. When the test length was increased to 40 items, this percentage increased to over 99% for all cells (See Table C.8). Sample size effects were no longer present with 40 items and beyond. Moreover, when test length was increased to 60 items (Table C.9), 100% of the replications in these cells were classified as having 2*r* dominant dimensions.

From these tables, it might seem that there is a different "rule" at work for simple structure data, other than *r+1*. However, consider a two dimensional, 20 item simple structure test, generated as it was here. Each of the 20 items has exactly one nonzero discrimination

parameter (split evenly between the two true dimensions of the test). If a PCA were conducted on the subset of items with nonzero discrimination parameters for the first dimension, the results would indicate *r+1* (i.e., 2) dominant dimensions. For the analogous subset for the second dimension, the results of a PCA would, again, result in 2 dominant dimensions (*r+1*). It follows that when the two subsets are analyzed together, the results indicate 2*r* dominant dimensions (i.e., *r+1* dominant dimensions for each of the true dimensions of the data). This is the case because it is as if there are two separate tests being analyzed, due to the simple structure items. Therefore, these results for simple structure data are still a form of the *r+1* principle. However, what is perplexing is the fact that with complex structure, the real dimensions share one artifactual dimension, yet with simple structure, each dimension is associated with its own artifactual dimension.

Beyond these interesting results, please note that there are two replications within Table C.8 that are labeled with "NA". This label denotes replications in which the number of dominant dimensions could not be determined. Further inspection of these replications revealed that, within one of the bootstrap replications or the simulated data itself, there was, by chance, at least one column (item) that had zero response variance. This made it impossible to create a correlation matrix and subsequently extract eigenvalues. This could prove to be problematic; however there were only 56 replications out of all 468,000 within the simulation study that were lost because of this anomaly (only 0.012%). Thus, this outcome isn't a general concern for the current study.

Three Dimensional Data

Complex Structure

The results for the three dimensional complex structure datasets are located in Tables C.10-C.12. For data with 20 items (Table C.10), the results show that for four and six response categories an overwhelming majority (>90%) of replications were classified as having *r+1* (i.e., 4) dominant dimensions. However, when examining binary data, there is more variance in dimensionality identification across replications. Nevertheless, the majority of replications within these cells was still *r+1*, albeit a smaller percentage (e.g., less than 50% when the sample size was 500). Based on the documented poor performance of PCA on binary data, this was expected. Additionally, when the data were binary, the other factors (sample size and correlation between dimensions) had a more pronounced effect (e.g., Table C.10). Indeed, the procedure identified four dominant dimensions for binary responses more than 80% of the time when sample $N \geq 1500$.

Moving forward, when test length is increased to 40 and 60 items respectively, the results seen in Tables C.11 and C.12 reflect those just described (from Table C.10). More specifically, the PCA continued to perform worse for binary data, but with four and six response categories the results show an equally high percentage (≈90% on average) of replications classified as *r+1*, where *r* is the true dimensionality of the data. Tables C.11 and C.12 show that the sample size effects seen for binary data were much less apparent when test length was increased beyond that (i.e., 40 and 60 items). Examining Table C.12 specifically, the bootstrap issues seen in two dimensional complex structure data were much more diminished here. Specifically, the tendency for the bootstrap to over extract dominant dimensions with 60 items was less of an issue with three dimensional, complex data.

Simple Structure

Lastly, the dominant dimension frequency counts for three dimensional simple structure datasets can be seen in Tables C.13-C.15. For tests of 20 items (Table C.13), there was no clear number of dominant dimensions. This was true for any of the cells represented in this table, even cells with 6 response categories and 2000 examinees. The inability for the PCA method to correctly identify the number of dominant dimensions in this case could be due to the fact that there were not enough items within these cells to adequately represent the dimensions of the test. The work by Roberts, Donoghue and Laughlin (2003) recommend 10-15 items for six response categories and one dimension. When considering a three dimensional simple structure test, a test length of 20 items would result in less than 7 items per dimension, by virtue of the way simple structure was achieved here (i.e., uniform assignment of items to dimensions). Consequently, the results for such cells in the simulation study were not as conclusive as those seen previously.

Proceeding to the next table (Table C.14) where test length is 40 items, the results show a high percentage (> 80%) of replications in each of these cells identified as having *2r* (i.e., 6) dominant dimensions, similar to the results from the two dimensional simple structure datasets. When the test length is further increased to 60 items, this conclusion (2*r*) is strengthened (See Table C.15). For all of these cells, over 97% of replications were classified as 2*r*.

Post hoc Analyses

All of the multidimensional tests previously presented were generated to have homogenous item structure. That is, for any simple structure test, all of the items on that test had simple structure, and the same was true for complex structure tests. However, the results illustrate that the PCA yielded two different conclusions, based on the structure of the test. Specifically, if the test had complex structure, the PCA yielded a decision of *r+1* dominant

dimensions, but if the test had simple structure, the PCA resulted in 2*r* dominant dimensions. Upon the discovery of this difference, post hoc analyses were conducted to investigate the performance of the PCA on datasets with heterogeneous item structure (i.e., mixed structure of simple and complex) in a small portion of conditions from the simulation study.

To accomplish this, data were generated to have two true dimensions. However, to achieve mixed structure, half of the items had simple structure, while the other half of the items had complex structure, following the procedures detailed earlier. The resulting datasets had 2000 examinees and zero correlation between the two dimensions. These specific levels of sample size and correlation between dimensions were fully crossed with all levels of the response category and test length factors. As with the full simulation study, 1,000 replications of each of the nine cells were generated. For each cell in this mixed structure condition, dimensionality was assessed in the same way as it was in the full simulation study. The results from the post hoc analyses can be found in Table C.16.

As seen in Table C.16, the majority of replications within each of the nine cells exhibited 2*r* (4) dominant dimensions. The percentage of replications for which dimensionality was classified as 2*r* was lower for datasets with only two response categories (56-70%), further illustrating poor performance of the PCA with binary data. Similar to the trend seen in previous results, increasing the response categories to four and then to six increased this percentage to above 99%. Based on these results, it seems that a PCA of two dimensional mixed structure tests with exactly half of the test having simple structure will yield 2*r* dominant dimensions, where *r* is the number of true dimensions.

Based on these results, one might hypothesize that, when the test structure is mixed between simple and complex, the PCA will indicate maximum number of dimensions out of the number of dimensions that are required for each subset of items, where subsets are formed on the basis of dimensional structure. For example, in the previous post hoc analysis, a PCA would identify three ($r+1$) dominant dimensions for the complex structure items (alone), and four ($2r$) dominant dimensions for the simple structure items. Then, when analyzed together, the PCA indicates four dominant dimensions; that is, maximum number of dominant dimensions out of three and four, required for complex and simple structure subsets of items, respectively. An additional set of post hoc analyses was done to further test this conclusion.

For this set of analyses, data were generated to have three true dimensions. However, to achieve mixed structure, half of the items had complex dimensional structure whereas the other half had simple structure. Within the set of simple structure items, half measured the first dimension whereas the other half measured the second, but none of these items measured the third dimension. The items with complex structure possessed nonzero item discriminations for all three dimensions. The datasets generated for this scenario had 2000 examinees, 40 items, six response categories and zero correlation between the two dimensions. As with the full simulation study, 1,000 replications were generated and dimensionality was assessed in the same way as it was in the full simulation study. The results from the post hoc analyses can be found in Table C.17.

Overwhelmingly, the results shown in Table C.17 show that the PCA method identified five dominant dimensions. This runs contrary to the previous hypothesis about a PCA of mixed structure data. For these data, if the above conclusion held, it would be expected that the PCA would identify four dominant dimensions, the maximum out of four ($r+1$) for the complex items

and four (2*r*) for the simple structure items. However, the results show that for this type of mixed structure there are five dominant dimensions. Therefore, the previous conclusion about mixed structure datasets needs to be amended. It seems that, in fact, a PCA of mixed structure data will result in the number of true dimensions plus the maximum number of artificial dimensions required for each of the item structure subsets. For example, in the preceding post hoc analysis, a PCA would identify four (*r+1*) dominant dimensions for the complex structure items (alone), with one artificial dimension, and four (2*r*) dominant dimensions for the simple structure items (alone), with two artificial dimensions. Then, when analyzed together, the PCA indicated five dominant dimensions; that is, three true dimensions plus two artificial dimensions (the maximum number of artificial dimensions for the complex and simple structure subsets, respectively).

CHAPTER 4

DISCUSSION

Outside of binary data, a PCA on item responses generated using the MGGUM resulted in *r+1* dominant dimensions a high proportion of the time when all items had complex structure. Furthermore, the simulation study showed that when all of data generated by the MGGUM had simple structure, a PCA of the corresponding item responses exhibited 2*r* dominant dimensions. These two conclusions were reliable under a variety of conditions. For tests with four or six response categories and at least 40 items, the correlation between dimensions and sample size had very minimal effects on performance, and the accuracy of this procedure remained at a high level. Additionally, post hoc analyses suggested that if the test data are evenly mixed with complex and simple structure items representing the same *r* dimensions, then a PCA will find the number of true dimensions plus the maximum number of artificial dimensions needed for each of the structure subsets.

While there is substantial research within the area of dimensionality assessment of cumulative IRT models, there has been very little attention paid to dimensionality assessment for unfolding IRT response data. Whereas, in the unfolding multidimensional scaling literature, Ross and Cliff (1964) have proven the *r+1* conjecture proposed by Coombs and Kao (1960), it does not necessarily hold for unfolding IRT. Of the small amount of research conducted on dimensionality assessment with unfolding IRT data, several researchers have commented on the presence of an "extra factor" when PCA is conducted on such data, but these researchers haven't seriously investigated it's functionality beyond the unidimensional case. The results of the current study verify that a PCA combined with the *r+1* rule is a viable dimensionality

assessment tool under a variety of conditions where data are generated with the MGGUM and items have complex structure and more than two response categories. Additionally, the results of this study establish the existence of a different, but consistent, rule for dimensionality assessment for simple structure data generated with the MGGUM; namely, there should be 2*r* dominant dimensions, where *r* is the true dimensionality of the test.

Beyond correct identification of dimensionality, this procedure was able to approximately recover the correct item assignment (location) to dimensions. Appendix E provides contains rank order correlations between true item locations (i.e., True δ_{i1}) and estimated item locations (i.e., eigenvectors) for prototypical replications within several conditions in the simulation study. Moderate to large correlations are observed between true and estimated item locations within these tables. Moreover, for simple structure conditions, the difference between a correlation of a true item location and the two corresponding estimated item locations is larger than for the complex conditions, as it should be because the items within simple structure conditions had only one true nonzero item discrimination, whereas all item discriminations were nonzero for complex structure conditions. Thus, conducting a PCA on unfolding data generated with the MGGUM not only will indicate dimensionality, but will also correctly assign items to dimensions.

Some limitations to the practicality of these results merit comment, as well as guidance on implementation. Initially, practitioners will need to verify that the data do, in fact, follow an ideal point process (i.e., are unfolding). Assuming this knowledge isn't apparent from theory, there are patterns that emerge in plots of the PCA results as a function of the ideal point process that can serve as an indicator. Consider two-dimensional simple structure item responses that

result from an ideal point process. Also, assume that the two dimensions are uncorrelated. Appendix F displays bivariate plots of the typical items between each pair of dominant eigenvectors in this scenario. Given that the data have simple structure, there will be 2*r* dominant dimensions, such that there are two eigenvectors that correspond to the "true" dimensions of the data and two that correspond to the artificial dimensions (i.e., the "plus one" for each dimension). When plotting the two estimated true dimensions against each other, the items will form a cross (See Figure F.1). However, when a true dimension is plotted against its corresponding artificial dimension, the items will form an arch (See Figures F.3 and F.4). When plotting a true dimension against an artificial dimension that is not its own, the items will form what looks like a perpendicular symbol seen in mathematics (⊥) (See Figures F.2 and F.5). Additionally, as seen in Figure F.6, when the two artificial dimensions are plotted against each other, the items will form an "L" shape, due to the fact the values on each of these artificial dimensions will be generally positive. When the data have simple structure, this arch shape can be gleaned easily from a two-dimensional plot (e.g., Figures E.3 and E.4). An analogous shape would be present for data with complex structure with two real dimensions, albeit in a three-dimensional space. It is difficult to portray the necessary three-dimensional plot. However, a three dimensional scatterplot has been generated in an attempt to illustrate this shape. Figures F.7 and F.8 show plot the three dominant eigenvectors from a PCA of two dimensional complex structure data. In these figures, the semblance of an arching surface (i.e., an upside down bowl) can be seen. In either case, the emergence of this shape indicates the use of a linear model (PCA) on nonlinear data (i.e., unfolding). Therefore, plotting the PCA results in this way can serve as identification tool for unfolding data.

Additionally, a second limitation to these results stems from the conditional choice between the *r+1* and the 2*r* rules, combined with the results of a PCA. Correct selection of the two rules is dependent on the knowledge of the test's structure (i.e., complex or simple structure items). In most cases, practitioners do not possess explicit knowledge of the test's structure. This leaves researchers at an impasse. However, there are still recommendations for utilization of this procedure in applied contexts.

For example, suppose a PCA is conducted on data that result from an ideal point process. If the results of the PCA indicate two or three dominant dimensions, it will be easy to decide on the correct dimensionality of the data. If the PCA identifies two dominant dimensions, the only choice is to employ the *r+1* rule (i.e., *r+1*=2*r=2*) and therefore the data are one-dimensional. Similarly, if the results of the analysis indicate three dominant dimensions, the only choice for dimensionality of the data is two—three can't equal 2*r*, and even if the structure is mixed, the rule suggested by the aforementioned post hoc analyses would still imply two dimensions.

Now, consider an analysis that results in four dominant dimensions. In this case, if the item structure is homogenous, four dominant dimensions could equal 2*r*, where *r* is equal to two, or four dominant dimensions could equal *r+1*, where *r* is three. Additionally, if the structure is mixed, based on the post hoc analyses, it could equal the number of true dimensions plus the maximum number of artificial dimensions needed for each of the structure subsets. Consequently, if the structure is mixed and the PCA has indicated four dominant dimensions, this could mean *r* is equal to two if there are simple structure items measuring two dimensions and complex structure items measuring those same two dimensions. Alternatively, if the structure is mixed, four dominant dimensions could mean *r* is equal to three if there are simple structure items that measure only one dimension and complex structure items measuring three

dimensions. Taking all of these scenarios into account, the choice for the dimensionality of the data is two or three. There are two ways in which the dimensionality can be chosen incorrectly in this case; namely, overestimation or underestimation by one. If the dimensionality is underestimated, and then the parameters in the MGGUM are estimated from the data, there will be a local dependence between items (i.e., a direct violation of a basic assumption of IRT models). On the other hand, if the dimensionality is overestimated, ad hoc studies in the unidimensional case of the MGGUM suggest that the item locations will exhibit an arch effect similar to that seen in correspondence analysis. In these cases, plotting the item locations may suggest that the dimensionality has been overestimated. Therefore, when in doubt, the recommendation is to overestimate the dimensionality to avoid local item dependence problems, and attempt to detect the overestimation after fitting a suitable latent trait model.

As a reminder, these recommendations have only been discussed for PCA results that result in two, three or four dominant dimensions. As r increases, and by definition the dominant dimensions resulting from the PCA increase, the strategy described above becomes problematic, because the different conclusions of true dimensionality become too discrepant. For example, even in the simple case of homogenous item structure, PCA results of eight dominant dimensions might indicate four ($2r$) dimensional data or seven ($r+1$) dimensional data.

Despite these limitations, the current study has successfully investigated and verified how a PCA analysis of unfolding response data generated with the MGGUM functions when the test data have homogenous item structure (i.e., simple or complex). As has been noted, the current study satisfies the need for a more robust dimensionality assessment tool for unfolding data, and even though the performance of this procedure has proven to be strong only with homogeneous item structure, its performance and potential still exceeds that of the other attempts in this

domain. To flesh out the rest of this story, more research is needed to explore performance of this procedure in a wider variety of mixed structure cases. Additionally, future research should examine the viability of detecting overestimation of the dimensionality after estimation using the MGGUM.

APPENDIX A

CUTOFF=MEAN

Table A.1

Dominant dimensions frequency counts for 1-dimensional 20 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note*:* Each column represents a given response category sample size combination.

Table A.2

Dominant dimensions frequency counts for 1-dimensional 40 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note*:* Each column represents a given response category sample size combination.

Table A.3

Dominant dimensions frequency counts for 1-dimensional 60 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note*:* Each column represents a given response category sample size combination.

Table A.4

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	24	0	0	0	0	0	0
3	988	990	972	1000	1000	1000	1000	1000	1000
4	12	10	4	0	0	0	0	0	0
N=1000									
2	0	0	10	0	0	0	0	0	0
3	996	1000	988	999	999	1000	999	1000	1000
4	4	0	2	1	1	0	1	0	0
N=1500									
2	0	0	5	0	0	0	0	0	0
3	997	996	995	999	999	1000	1000	1000	1000
4	3	4	0	1	1	0	0	0	0
N=2000									
2	0	0	2	0	0	0	0	0	0
3	997	997	998	998	1000	1000	998	997	1000
4	3	3	0	2	0	0	2	3	0

Note*:* Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.5

Dominant dimensions frequency counts for 2-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	952	959	999	991	994	1000	984	983	1000
4	48	41	1	9	6	0	16	17	0
N=1000									
3	904	926	996	977	975	1000	932	934	994
4	96	74	4	23	25	0	68	66	6
N=1500									
3	839	864	982	953	953	997	897	903	989
4	161	136	18	47	47	3	103	97	11
N=2000									
3	795	817	985	916	915	992	880	892	989
4	205	183	15	84	85	8	120	108	11

Note*:* Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.6

Dominant dimensions frequency counts for 2-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	782	802	967	958	952	1000	853	864	989
4	218	198	33	42	48	0	147	136	11
N=1000									
3	568	591	930	817	808	990	576	616	933
4	432	409	70	183	192	10	424	384	67
N=1500									
3	403	399	866	705	697	963	428	461	872
4	597	601	134	295	303	37	572	539	128
N=2000									
3	281	304	786	592	591	933	335	344	803
4	719	696	214	408	409	67	665	656	197

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.7

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	4	4	3	6	8	6	1	2	2
3	78	78	157	141	129	241	115	108	179
4	918	918	840	853	863	753	884	890	819
N=1000									
2	2	2	0	4	5	4	1	2	1
3	46	37	77	84	78	162	63	60	118
4	952	961	923	912	917	834	936	938	881
N=1500									
2	1	1	0	4	2	2	1	1	1
3	32	28	63	61	66	123	55	52	99
4	967	971	937	935	932	875	944	947	900
N=2000									
2	0	0	0	3	3	1	1	1	0
3	26	26	52	64	65	117	53	58	90
4	974	974	948	933	932	882	946	941	910

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.8

Dominant dimension frequency counts for 2-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	0	0	0	2	0	0	0
4	1000	1000	1000	1000	1000	997	1000	1000	1000
NA	0	0	0	0	0	1	0	0	0
N=1000									
3	0	0	0	0	0	0	1	1	1
4	1000	1000	1000	1000	997	1000	999	999	999
NA	0	0	0	0	3	0	0	0	0
N=1500									
3	0	0	0	0	0	0	1	0	1
4	1000	1000	1000	1000	1000	1000	999	1000	999
N=2000									
4	1000	1000	1000	999	1000	1000	1000	1000	1000
NA	0	0	0	1	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.9

Dominant dimension frequency counts for 2-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=2000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.10

Dominant dimension frequency counts for 3-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	2	0	230	0	0	0	0	0	3
3	81	103	415	10	9	179	7	4	216
4	455	463	251	990	990	819	993	996	781
5	316	310	83	0	1	2	0	0	0
6	104	94	17	0	0	0	0	0	0
7	30	22	4	0	0	0	0	0	0
8	4	4	0	0	0	0	0	0	0
9	1	1	0	0	0	0	0	0	0
10	0	1	0	0	0	0	0	0	0
NA	7	2	0	0	0	0	0	0	0
N=1000									
2	0	1	176	0	0	0	0	0	0
3	52	65	559	3	4	68	1	0	124
4	706	707	234	997	996	932	999	1000	876
5	217	197	31	0	0	0	0	0	0
6	22	30	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0
N=1500									
2	0	0	131	0	0	0	0	0	0
3	49	59	624	0	1	50	2	2	80
4	804	822	235	1000	999	950	998	998	920
5	142	108	10	0	0	0	0	0	0
6	5	11	0	0	0	0	0	0	0
N=2000									
2	0	0	103	0	0	0	0	0	0
3	34	35	646	1	0	40	1	4	69
4	896	877	244	999	1000	960	999	996	931
5	65	86	7	0	0	0	0	0	0
6	5	2	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.11

Dominant dimension frequency counts for 3-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	2	0	0	0	0	0	0
3	1	0	126	0	0	0	0	0	0
4	180	175	405	999	1000	1000	994	995	1000
5	370	377	317	1	0	0	6	5	0
6	270	275	112	0	0	0	0	0	0
7	119	107	32	0	0	0	0	0	0
8	46	48	4	0	0	0	0	0	0
9	5	8	1	0	0	0	0	0	0
10	1	2	0	0	0	0	0	0	0
11	0	2	0	0	0	0	0	0	0
NA	8	6	1	0	0	0	0	0	0
N=1000									
2	0	0	2	0	0	0	0	0	0
3	0	0	92	0	0	0	0	0	0
4	400	412	639	1000	1000	1000	985	991	1000
5	429	435	233	0	0	0	15	9	0
6	151	132	30	0	0	0	0	0	0
7	16	18	4	0	0	0	0	0	0
8	3	3	0	0	0	0	0	0	0
NA	1	0	0	0	0	0	0	0	0
N=1500									
3	0	0	53	0	0	0	0	0	0
4	541	565	809	1000	1000	1000	979	979	541
5	389	381	133	0	0	0	21	21	389
6	66	53	5	0	0	0	0	0	66
7	4	1	0	0	0	0	0	0	4
N=2000									
3	0	0	26	0	0	0	0	0	0
4	609	612	855	1000	1000	1000	967	969	609
5	368	357	114	0	0	0	33	31	368
6	23	31	5	0	0	0	0	0	23

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.12

Dominant dimension frequency counts for 3-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	7	0	0	0	0	0	0
4	62	60	163	1000	1000	1000	963	962	999
5	227	234	338	0	0	0	37	38	1
6	304	331	286	0	0	0	0	0	0
7	223	207	140	0	0	0	0	0	0
8	110	103	53	0	0	0	0	0	0
9	41	39	11	0	0	0	0	0	0
10	18	14	0	0	0	0	0	0	0
11	1	3	1	0	0	0	0	0	0
NA	14	9	1	0	0	0	0	0	0
N=1000									
3	0	0	2	0	0	0	0	0	0
4	161	153	420	999	1000	1000	853	858	996
5	444	443	397	1	0	0	146	142	4
6	297	298	151	0	0	0	1	0	0
7	76	87	28	0	0	0	0	0	0
8	17	15	1	0	0	0	0	0	0
9	3	3	1	0	0	0	0	0	0
10	0	1	0	0	0	0	0	0	0
NA	2	0	0	0	0	0	0	0	0
N=1500									
4	202	180	550	1000	999	1000	743	768	996
5	541	538	379	0	1	0	255	230	4
6	227	255	66	0	0	0	2	2	0
7	30	26	4	0	0	0	0	0	0
8	0	1	1	0	0	0	0	0	0
N=2000									
4	180	182	636	996	1000	1000	653	684	988
5	584	614	334	4	0	0	332	305	12
6	218	194	29	0	0	0	15	11	0
7	18	10	1	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.13

Dominant dimension frequency counts for 3-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	31	31	36	44	46	29	27	36	27
4	174	170	352	247	257	483	219	222	398
5	406	420	454	460	443	388	429	423	453
6	389	379	158	249	254	100	325	319	122
N=1000									
3	23	20	13	26	23	10	17	18	7
4	115	126	282	198	208	381	151	146	301
5	390	394	466	449	428	460	412	404	484
6	472	460	239	327	341	149	420	432	208
N=1500									
3	10	8	4	32	28	9	14	12	5
4	111	115	249	150	163	310	121	109	266
5	366	372	471	436	434	501	420	434	511
6	513	505	276	382	375	180	445	445	218
N=2000									
3	3	4	1	21	20	10	15	11	4
4	89	89	193	129	128	282	109	109	245
5	353	358	483	410	425	515	419	423	518
6	555	549	323	440	427	193	457	457	233

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.14

Dominant dimension frequency counts for 3-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1	0	3	4	9	18	1	4	8
5	44	41	127	134	127	241	76	72	158
6	955	959	870	862	864	741	923	924	833
NA	0	0	0	0	0	0	0	0	1
N=1000									
4	0	1	1	4	4	7	1	1	2
5	19	15	36	71	68	130	29	30	59
6	981	984	963	925	928	863	969	968	938
NA	0	0	0	0	0	0	1	1	1
N=1500									
4	0	0	1	1	1	1	1	0	2
5	14	11	27	55	51	85	29	31	53
6	986	989	972	944	948	914	969	969	942
NA	0	0	0	0	0	0	1	0	3
N=2000									
4	0	0	0	0	0	1	0	0	0
5	12	10	24	45	46	70	24	23	49
6	988	990	976	955	954	929	976	977	951

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.15

Dominant dimension frequency counts for 3-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
5	0	0	4	6	9	20	6	5	10
6	1000	1000	996	994	991	980	993	995	989
NA	0	0	0	0	0	0	1	0	1
N=1000									
5	0	0	0	0	2	7	1	1	4
6	1000	1000	1000	1000	998	993	999	999	996
N=1500									
5	0	0	0	0	0	4	0	0	3
6	1000	1000	1000	1000	1000	996	999	998	996
NA	0	0	0	0	0	0	1	2	1
N=2000									
5	0	0	0	1	1	3	0	0	1
6	1000	1000	1000	999	999	997	998	997	998
NA	0	0	0	0	0	0	2	3	1

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.16

Dominant dimension frequency counts for 2-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	2 RC			4 RC			6 RC		
N=2000	20	40	60	20	40	60	20	40	60
2	0	1	0	0	0	0	0	0	0
3	266	381	277	1	2	1	0	0	0
4	734	618	723	999	998	999	999	1000	1000
5	0	0	0	0	0	0	1	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table A.17

Dominant dimension frequency counts for 3-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	6 RC
N=2000	40
5	963
6	37

APPENDIX B

CUTOFF=75TH PERCENTILE

Table B.1

Dominant dimensions frequency counts for 1-dimensional 20 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table B.2

Dominant dimensions frequency counts for 1-dimensional 40 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table B.3

Dominant dimensions frequency counts for 1-dimensional 60 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table B.4

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	27	0	0	0	0	0	0
3	994	993	971	1000	1000	1000	1000	1000	1000
4	6	7	2	0	0	0	0	0	0
N=1000									
2	0	0	10	0	0	0	0	0	0
3	997	1000	990	1000	1000	1000	1000	1000	1000
4	3	0	0	0	0	0	0	0	0
N=1500									
2	0	0	6	0	0	0	0	0	0
3	998	997	994	999	999	1000	1000	1000	1000
4	2	3	0	1	1	0	0	0	0
N=2000									
2	0	0	2	0	0	0	0	0	0
3	998	998	998	998	1000	1000	999	998	1000
4	2	2	0	2	0	0	1	2	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.5

Dominant dimensions frequency counts for 2-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	960	971	1000	992	996	1000	986	987	1000
4	40	29	0	8	4	0	14	13	0
N=1000									
3	930	940	998	979	981	1000	942	941	995
4	70	60	2	21	19	0	58	59	5
N=1500									
3	859	883	988	961	958	998	905	912	992
4	141	117	12	39	42	2	95	88	8
N=2000									
3	823	838	987	924	922	994	887	896	991
4	177	162	13	76	78	6	113	104	9

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.6

Dominant dimensions frequency counts for 2-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	826	842	977	962	963	1000	870	879	990
4	174	158	23	38	37	0	130	121	10
N=1000									
3	612	629	947	835	834	994	602	638	940
4	388	371	53	165	166	6	398	362	60
N=1500									
3	436	436	887	730	727	968	451	484	887
4	564	564	113	270	273	32	549	516	113
N=2000									
3	315	323	807	606	609	938	359	369	822
4	685	677	193	394	391	62	641	631	178

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.7

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	6	5	3	10	9	7	2	2	2
3	87	91	175	154	146	255	127	120	201
4	907	904	822	836	845	738	871	878	797
N=1000									
2	3	2	0	4	6	4	1	2	1
3	49	42	92	92	81	175	66	63	127
4	948	956	908	904	913	821	933	935	872
N=1500									
2	1	1	0	4	2	2	1	1	1
3	33	34	65	66	71	131	56	58	104
4	966	965	935	930	927	867	943	941	895
N=2000									
2	0	0	0	3	3	1	1	1	0
3	28	29	54	65	70	122	55	60	94
4	972	971	946	932	927	877	944	939	906

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.8

Dominant dimension frequency counts for 2-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	0	1	1	2	0	0	1
4	1000	1000	1000	999	999	997	1000	1000	999
NA	0	0	0	0	0	1	0	0	0
N=1000									
3	0	0	0	0	0	0	1	1	1
4	1000	1000	1000	1000	997	1000	999	999	999
NA	0	0	0	0	3	0	0	0	0
N=1500									
3	0	0	0	0	0	0	1	0	1
4	1000	1000	1000	1000	1000	1000	999	1000	999
N=2000									
4	1000	1000	1000	999	1000	1000	1000	1000	1000
NA	0	0	0	1	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.9

Dominant dimension frequency counts for 2-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=2000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.10

Dominant dimension frequency counts for 3-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
1	0	0	1	0	0	0	0	0	0
2	4	3	315	0	0	2	0	0	4
3	131	158	438	28	28	284	12	8	294
4	521	528	189	972	972	714	988	992	702
5	267	256	49	0	0	0	0	0	0
6	58	44	7	0	0	0	0	0	0
7	9	9	1	0	0	0	0	0	0
8	2	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0
NA	7	2	0	0	0	0	0	0	0
N=1000									
2	0	1	229	0	0	0	0	0	1
3	76	93	563	7	7	110	1	2	161
4	765	750	192	993	993	890	999	998	838
5	142	142	16	0	0	0	0	0	0
6	15	14	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0
N=1500									
2	0	0	173	0	0	0	0	0	0
3	61	80	624	1	1	57	2	2	94
4	839	839	198	999	999	943	998	998	906
5	98	80	5	0	0	0	0	0	0
6	2	1	0	0	0	0	0	0	0
N=2000									
2	0	0	145	0	0	0	0	0	0
3	43	51	653	3	0	45	2	4	81
4	912	891	200	997	1000	955	998	996	919
5	43	56	2	0	0	0	0	0	0
6	2	2	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.11

Dominant dimension frequency counts for 3-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	6	0	0	0	0	0	0
3	2	0	181	0	0	0	0	0	0
4	271	261	459	1000	1000	1000	996	996	1000
5	409	429	273	0	0	0	4	4	0
6	213	228	59	0	0	0	0	0	0
7	75	56	19	0	0	0	0	0	0
8	20	15	1	0	0	0	0	0	0
9	2	3	1	0	0	0	0	0	0
10	0	1	0	0	0	0	0	0	0
11	0	1	0	0	0	0	0	0	0
NA	8	6	1	0	0	0	0	0	0
N=1000									
2	0	0	2	0	0	0	0	0	0
3	0	0	141	0	0	0	0	0	0
4	504	516	664	1000	1000	1000	989	991	1000
5	376	392	175	0	0	0	11	9	0
6	113	80	17	0	0	0	0	0	0
7	3	10	1	0	0	0	0	0	0
8	3	2	0	0	0	0	0	0	0
NA	1	0	0	0	0	0	0	0	0
N=1500									
3	0	0	69	0	0	0	0	0	0
4	622	653	833	1000	1000	1000	985	985	1000
5	340	316	95	0	0	0	15	15	0
6	36	30	3	0	0	0	0	0	0
7	2	1	0	0	0	0	0	0	0
N=2000									
3	0	0	35	0	0	0	0	0	0
4	676	667	876	1000	1000	1000	970	971	1000
5	309	310	85	0	0	0	30	29	0
6	15	23	4	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.12

Dominant dimension frequency counts for 3-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	12	0	0	0	0	0	0
4	104	116	237	1000	1000	1000	970	972	1000
5	295	292	384	0	0	0	30	28	0
6	310	324	237	0	0	0	0	0	0
7	186	174	99	0	0	0	0	0	0
8	64	64	25	0	0	0	0	0	0
9	23	16	5	0	0	0	0	0	0
10	4	5	0	0	0	0	0	0	0
NA	14	9	1	0	0	0	0	0	0
N=1000									
3	0	0	3	0	0	0	0	0	0
4	215	213	490	1000	1000	1000	872	875	996
5	478	471	376	0	0	0	127	125	4
6	243	259	113	0	0	0	1	0	0
7	52	48	17	0	0	0	0	0	0
8	10	8	1	0	0	0	0	0	0
9	0	1	0	0	0	0	0	0	0
NA	2	0	0	0	0	0	0	0	0
N=1500									
4	257	240	625	1000	999	1000	764	791	999
5	547	543	336	0	1	0	234	208	1
6	176	204	38	0	0	0	2	1	0
7	20	12	1	0	0	0	0	0	0
8	0	1	0	0	0	0	0	0	0
N=2000									
4	236	241	692	997	1000	1000	681	707	990
5	579	599	289	3	0	0	305	285	10
6	177	153	18	0	0	0	14	8	0
7	8	7	1	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.13

Dominant dimension frequency counts for 3-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	37	38	38	56	62	34	34	45	27
4	192	182	399	258	266	509	231	235	436
5	418	437	434	457	436	372	435	431	432
6	353	343	129	229	236	85	300	289	105
N=1000									
3	27	26	17	28	27	13	18	18	8
4	121	135	302	215	220	403	169	156	322
5	399	403	470	451	430	445	409	409	484
6	453	436	211	306	323	139	404	417	186
N=1500									
3	11	12	5	36	33	9	17	13	5
4	121	125	258	164	168	337	126	125	285
5	380	374	476	429	432	487	426	433	504
6	488	489	261	371	367	167	431	429	206
N=2000									
3	4	5	1	22	21	13	16	14	4
4	102	96	206	143	138	298	113	115	255
5	351	363	484	411	424	510	423	425	524
6	543	536	309	424	417	179	448	446	217

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.14

Dominant dimension frequency counts for 3-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1	0	3	5	10	21	2	4	9
5	48	47	142	141	135	249	79	80	170
6	951	953	855	854	855	730	919	916	820
NA	0	0	0	0	0	0	0	0	1
N=1000									
4	0	1	1	4	5	7	1	1	2
5	20	17	43	77	69	136	29	33	65
6	980	982	956	919	926	857	969	965	932
NA	0	0	0	0	0	0	1	1	1
N=1500									
4	0	0	1	1	2	1	1	1	2
5	14	12	32	63	55	92	30	30	57
6	986	988	967	936	943	907	968	969	938
NA	0	0	0	0	0	0	1	0	3
N=2000									
4	0	0	0	0	0	1	0	0	0
5	14	11	24	45	46	77	27	24	50
6	986	989	976	955	954	922	973	976	950

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.15

Dominant dimension frequency counts for 3-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	0	0	0	0	0	1	0	0	0
5	0	0	5	9	10	23	6	5	12
6	1000	1000	995	991	990	976	993	995	987
NA	0	0	0	0	0	0	1	0	1
N=1000									
5	0	0	0	0	2	7	2	2	5
6	1000	1000	1000	1000	998	993	998	998	995
N=1500									
5	0	0	0	0	0	4	0	0	3
6	1000	1000	1000	1000	1000	996	999	998	996
NA	0	0	0	0	0	0	1	2	1
N=2000									
5	0	0	0	1	1	3	0	0	1
6	1000	1000	1000	999	999	997	998	997	998
NA	0	0	0	0	0	0	2	3	1

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.16

Dominant dimension frequency counts for 2-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	2 RC			4 RC			6 RC		
N=2000	20	40	60	20	40	60	20	40	60
2	0	1	0	0	0	0	0	0	0
3	283	399	288	1	4	1	0	0	0
4	717	600	712	999	996	999	999	1000	1000
5	0	0	0	0	0	0	1	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table B.17

Dominant dimension frequency counts for 3-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	6 RC
N=2000	40
5	966
6	34

APPENDIX C

CUTOFF=95TH PERCENTILE

Table C.1

Dominant dimensions frequency counts for 1-dimensional 20 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table C.2

Dominant dimensions frequency counts for 1-dimensional 40 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table C.3

Dominant dimensions frequency counts for 1-dimensional 60 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table C.4

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	47	0	0	2	0	0	0
3	997	996	953	1000	1000	998	1000	1000	1000
4	3	4	0	0	0	0	0	0	0
N=1000									
2	0	0	14	0	0	0	0	0	0
3	999	1000	986	1000	1000	1000	1000	1000	1000
4	1	0	0	0	0	0	0	0	0
N=1500									
2	0	0	8	0	0	0	0	0	0
3	998	998	992	1000	1000	1000	1000	1000	1000
4	2	2	0	0	0	0	0	0	0
N=2000									
2	0	0	3	0	0	0	0	0	0
3	998	999	997	998	1000	1000	999	998	1000
4	2	1	0	2	0	0	1	2	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.5

Dominant dimensions frequency counts for 2-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	976	981	1000	995	998	1000	989	991	1000
4	24	19	0	5	2	0	11	9	0
N=1000									
3	953	958	1000	985	986	1000	955	951	996
4	47	42	0	15	14	0	45	49	4
N=1500									
3	885	915	993	967	965	998	918	923	995
4	115	85	7	33	35	2	82	77	5
N=2000									
3	855	866	991	937	938	996	901	911	992
4	145	134	9	63	62	4	99	89	8

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.6

Dominant dimensions frequency counts for 2-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	867	880	988	971	978	1000	894	905	993
4	133	120	12	29	22	0	106	95	7
N=1000									
3	665	695	959	861	867	996	635	674	949
4	335	305	41	139	133	4	365	326	51
N=1500									
3	496	491	913	751	754	974	488	514	905
4	504	509	87	249	246	26	512	486	95
N=2000									
3	357	377	837	646	649	951	386	397	842
4	643	623	163	354	351	49	614	603	158

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.7

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	7	7	4	14	12	8	5	5	2
3	105	106	210	168	168	287	141	136	231
4	888	887	786	818	820	705	854	859	767
N=1000									
2	4	2	1	6	6	6	1	3	1
3	54	51	105	99	92	197	78	72	141
4	942	947	894	895	902	797	921	925	858
N=1500									
2	1	1	0	4	3	2	1	1	1
3	38	34	74	75	75	144	61	67	119
4	961	965	926	921	922	854	938	932	880
N=2000									
2	0	0	0	3	3	1	2	1	0
3	32	34	59	69	74	134	61	62	107
4	968	966	941	928	923	865	937	937	893

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.8

Dominant dimension frequency counts for 2-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	0	1	1	4	0	0	2
4	1000	1000	1000	999	999	995	1000	1000	998
NA	0	0	0	0	0	1	0	0	0
N=1000									
3	0	0	0	0	0	0	1	1	1
4	1000	1000	1000	1000	997	1000	999	999	999
NA	0	0	0	0	3	0	0	0	0
N=1500									
3	0	0	0	0	0	0	1	0	1
4	1000	1000	1000	1000	1000	1000	999	1000	999
N=2000									
4	1000	1000	1000	999	1000	1000	1000	1000	1000
NA	0	0	0	1	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.9

Dominant dimension frequency counts for 2-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=2000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.10

Dominant dimension frequency counts for 3-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
1	0	0	1	0	0	0	0	0	0
2	18	12	452	0	0	2	0	0	4
3	258	274	416	28	28	284	12	8	294
4	561	573	112	972	972	714	988	992	702
5	137	123	18	0	0	0	0	0	0
6	17	15	1	0	0	0	0	0	0
7	1	1	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0
NA	7	2	0	0	0	0	0	0	0
N=1000									
2	1	1	410	0	0	0	0	0	1
3	137	155	503	7	7	110	1	2	161
4	774	760	84	993	993	890	999	998	838
5	82	77	3	0	0	0	0	0	0
6	5	7	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0
N=1500									
2	0	0	255	0	0	0	0	0	0
3	84	111	599	2	3	79	2	5	114
4	869	844	145	998	997	921	998	995	886
5	46	45	1	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0
N=2000									
2	1	2	204	0	0	0	0	0	0
3	62	81	649	3	0	50	3	4	96
4	911	887	147	997	1000	950	997	996	904
5	26	29	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.11

Dominant dimension frequency counts for 3-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	21	0	0	0	0	0	0
3	4	3	286	0	0	0	0	0	0
4	456	430	479	1000	1000	1000	997	998	1000
5	351	397	183	0	0	0	3	2	0
6	139	140	24	0	0	0	0	0	0
7	37	17	6	0	0	0	0	0	0
8	4	6	0	0	0	0	0	0	0
9	1	1	0	0	0	0	0	0	0
NA	8	6	1	0	0	0	0	0	0
N=1000									
2	0	0	3	0	0	0	0	0	0
3	0	0	199	0	0	0	0	0	0
4	642	645	689	1000	1000	1000	990	995	1000
5	307	310	100	0	0	0	10	5	0
6	46	42	9	0	0	0	0	0	0
7	3	3	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0
NA	1	0	0	0	0	0	0	0	0
N=1500									
3	0	0	112	0	0	0	0	0	0
4	730	746	834	1000	1000	1000	989	988	1000
5	256	238	53	0	0	0	11	12	0
6	14	16	1	0	0	0	0	0	0
N=2000									
3	0	0	53	0	0	0	0	0	0
4	769	761	896	1000	1000	1000	973	976	1000
5	228	224	51	0	0	0	27	24	0
6	3	15	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.12

Dominant dimension frequency counts for 3-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	32	0	0	0	0	0	0
4	204	205	354	1000	1000	1000	980	982	1000
5	366	385	383	0	0	0	20	18	0
6	265	267	175	0	0	0	0	0	0
7	107	104	42	0	0	0	0	0	0
8	33	24	10	0	0	0	0	0	0
9	9	6	3	0	0	0	0	0	0
10	2	0	0	0	0	0	0	0	0
NA	14	9	1	0	0	0	0	0	0
N=1000									
3	0	0	7	0	0	0	0	0	0
4	326	323	628	1000	1000	1000	897	901	999
5	490	485	300	0	0	0	102	99	1
6	162	170	61	0	0	0	1	0	0
7	18	20	3	0	0	0	0	0	0
8	2	1	1	0	0	0	0	0	0
9	0	1	0	0	0	0	0	0	0
NA	2	0	0	0	0	0	0	0	0
N=1500									
4	369	351	735	1000	999	1000	804	826	999
5	517	534	246	0	1	0	195	174	1
6	107	111	18	0	0	0	1	0	0
7	7	4	1	0	0	0	0	0	0
N=2000									
4	323	321	777	999	1000	1000	722	742	991
5	561	571	216	1	0	0	266	251	9
6	115	104	7	0	0	0	12	7	0
7	1	4	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.13

Dominant dimension frequency counts for 3-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	45	53	48	73	79	40	49	50	34
4	220	214	463	286	291	551	241	257	486
5	422	438	393	428	424	341	442	436	396
6	313	295	96	213	206	68	268	257	84
N=1000									
3	30	31	20	37	37	19	21	22	13
4	134	152	335	230	241	431	185	176	355
5	411	406	461	443	428	424	419	414	460
6	425	411	184	290	294	126	375	388	172
N=1500									
3	15	16	7	40	39	11	18	17	6
4	134	130	283	182	175	375	137	139	319
5	386	388	479	426	438	465	428	436	481
6	465	466	231	352	348	149	417	408	194
N=2000									
3	6	6	2	28	27	14	17	16	4
4	111	106	228	157	152	319	124	128	282
5	358	375	484	411	420	501	428	426	511
6	525	513	286	404	401	166	431	430	203

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.14

Dominant dimension frequency counts for 3-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	2	0	4	6	10	24	3	5	11
5	60	63	165	152	150	266	89	91	186
6	938	937	831	842	840	710	908	904	802
NA	0	0	0	0	0	0	0	0	1
N=1000									
4	0	1	2	4	5	8	1	1	2
5	23	22	51	82	75	146	33	34	74
6	977	977	947	914	920	846	965	964	923
NA	0	0	0	0	0	0	1	1	1
N=1500									
4	0	0	1	1	2	2	1	1	2
5	16	15	36	68	58	101	32	34	61
6	984	985	963	931	940	897	966	965	934
NA	0	0	0	0	0	0	1	0	3
N=2000									
4	0	0	0	0	0	1	0	0	1
5	15	11	27	50	47	85	29	25	52
6	985	989	973	950	953	914	971	975	947

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.15

Dominant dimension frequency counts for 3-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	0	0	0	0	0	1	0	0	0
5	0	0	9	11	11	29	7	5	13
6	1000	1000	991	989	989	970	992	995	986
NA	0	0	0	0	0	0	1	0	1
N=1000									
5	0	0	0	2	2	8	2	2	5
6	1000	1000	1000	998	998	992	998	998	995
N=1500									
5	0	0	0	0	1	4	0	0	3
6	1000	1000	1000	1000	999	996	999	998	996
NA	0	0	0	0	0	0	1	2	1
N=2000									
5	0	0	0	1	1	4	0	0	1
6	1000	1000	1000	999	999	996	998	997	998
NA	0	0	0	0	0	0	2	3	1

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.16

Dominant dimension frequency counts for 2-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	2 RC			4 RC			6 RC		
N=2000	20	40	60	20	40	60	20	40	60
2	0	1	0	0	0	0	0	0	0
3	312	431	311	1	5	2	0	0	0
4	688	568	689	999	995	998	999	1000	1000
5	0	0	0	0	0	0	1	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table C.17

Dominant dimension frequency counts for 3-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	6 RC
N=2000	40
5	972
6	28

APPENDIX D

CUTOFF=99TH PERCENTILE

Table D.1

Dominant dimensions frequency counts for 1-dimensional 20 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table D.2

Dominant dimensions frequency counts for 1-dimensional 40 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table D.3

Dominant dimensions frequency counts for 1-dimensional 60 item datasets.

	500			1000			1500			2000		
Dominant Dimensions	2	4	6	2	4	6	2	4	6	2	4	6
2	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category sample size combination.

Table D.4

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	64	0	0	2	0	0	0
3	998	1000	936	1000	1000	998	1000	1000	1000
4	2	0	0	0	0	0	0	0	0
N=1000									
2	0	0	19	0	0	0	0	0	0
3	1000	1000	981	1000	1000	1000	1000	1000	1000
4	0	0	0	0	0	0	0	0	0
N=1500									
2	0	0	9	0	0	0	0	0	0
3	998	999	991	1000	1000	1000	1000	1000	1000
4	2	1	0	0	0	0	0	0	0
N=2000									
2	0	0	5	0	0	0	0	0	0
3	999	999	995	998	1000	1000	1000	1000	1000
4	1	1	0	2	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.5

Dominant dimensions frequency counts for 2-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	981	987	1000	997	999	1000	991	993	1000
4	19	13	0	3	1	0	9	7	0
N=1000									
3	967	972	1000	990	989	1000	961	955	997
4	33	28	0	10	11	0	39	45	3
N=1500									
3	905	934	994	974	974	999	930	933	996
4	95	66	6	26	26	1	70	67	4
N=2000									
3	869	889	993	943	944	996	906	922	994
4	131	111	7	57	56	4	94	78	6

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.6

Dominant dimensions frequency counts for 2-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	897	907	991	982	984	1000	911	911	995
4	103	93	9	18	16	0	89	89	5
N=1000									
3	710	727	969	882	879	996	662	690	953
4	290	273	31	118	121	4	338	310	47
N=1500									
3	525	528	926	770	779	979	519	542	920
4	475	472	74	230	221	21	481	458	80
N=2000									
3	393	415	855	665	670	960	405	421	856
4	607	585	145	335	330	40	595	579	144

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.7

Dominant dimensions frequency counts for 2-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	9	7	4	16	12	8	6	5	2
3	117	129	239	189	188	311	150	151	252
4	874	864	757	795	800	681	844	844	746
N=1000									
2	4	3	1	7	6	6	1	3	1
3	55	58	119	109	102	213	89	83	152
4	941	939	880	884	892	781	910	914	847
N=1500									
2	1	1	0	5	5	2	2	1	1
3	41	36	80	86	82	160	65	73	127
4	958	963	920	909	913	838	933	926	872
N=2000									
2	0	0	0	3	3	2	3	2	1
3	36	37	65	72	79	143	64	65	111
4	964	963	935	925	918	855	933	933	888

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.8

Dominant dimension frequency counts for 2-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	0	0	0	1	2	4	0	0	2
4	1000	1000	1000	999	998	995	1000	1000	998
NA	0	0	0	0	0	1	0	0	0
N=1000									
3	0	0	0	0	0	0	1	1	1
4	1000	1000	1000	1000	997	1000	999	999	999
NA	0	0	0	0	3	0	0	0	0
N=1500									
3	0	0	0	0	0	0	1	0	1
4	1000	1000	1000	1000	1000	1000	999	1000	999
N=2000									
3	0	0	0	0	0	0	0	0	0
4	1000	1000	1000	999	1000	1000	1000	1000	1000
NA	0	0	0	1	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.9

Dominant dimension frequency counts for 2-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=1500									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000
N=2000									
4	1000	1000	1000	1000	1000	1000	1000	1000	1000

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.10

Dominant dimension frequency counts for 3-dimensional 20 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
1	0	0	2	0	0	0	0	0	0
2	29	34	556	0	0	4	0	0	9
3	353	364	359	40	43	343	13	11	329
4	521	527	77	960	957	653	987	989	662
5	85	68	6	0	0	0	0	0	0
6	4	4	0	0	0	0	0	0	0
7	1	1	0	0	0	0	0	0	0
NA	7	2	0	0	0	0	0	0	0
N=1000									
2	1	1	410	0	0	0	0	0	1
3	182	214	503	8	8	125	1	6	183
4	770	741	84	992	992	875	999	994	816
5	45	44	3	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0
N=1500									
2	0	0	317	0	0	0	0	0	0
3	108	144	573	3	3	91	3	5	125
4	868	828	110	997	997	909	997	995	875
5	24	28	0	0	0	0	0	0	0
N=2000									
2	1	2	254	0	0	0	0	0	0
3	75	106	630	3	1	61	3	4	104
4	914	875	116	997	999	939	997	996	896
5	10	16	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.11

Dominant dimension frequency counts for 3-dimensional 40 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	44	0	0	0	0	0	0
3	7	5	355	0	0	0	0	0	0
4	565	556	458	1000	1000	1000	999	999	1000
5	304	344	127	0	0	0	1	1	0
6	94	75	13	0	0	0	0	0	0
7	22	10	2	0	0	0	0	0	0
8	0	3	0	0	0	0	0	0	0
9	0	1	0	0	0	0	0	0	0
NA	8	6	1	0	0	0	0	0	0
N=1000									
2	0	0	7	0	0	0	0	0	0
3	0	0	235	0	0	0	0	0	0
4	734	740	690	1000	1000	1000	990	996	1000
5	236	236	65	0	0	0	10	4	0
6	28	22	3	0	0	0	0	0	0
7	1	2	0	0	0	0	0	0	0
NA	1	0	0	0	0	0	0	0	0
N=1500									
3	0	0	135	0	0	0	0	0	0
4	795	807	833	1000	1000	1000	991	993	1000
5	197	189	32	0	0	0	9	7	0
6	8	4	0	0	0	0	0	0	0
N=2000									
3	0	0	74	0	0	0	0	0	0
4	815	807	890	1000	1000	1000	978	985	1000
5	184	183	36	0	0	0	22	15	0
6	1	10	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.12

Dominant dimension frequency counts for 3-dimensional 60 item datasets, complex structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
2	0	0	1	0	0	0	0	0	0
3	0	0	68	0	0	0	0	0	0
4	281	310	439	1000	1000	1000	984	987	1000
5	418	411	348	0	0	0	16	13	0
6	212	192	118	0	0	0	0	0	0
7	56	66	20	0	0	0	0	0	0
8	15	9	5	0	0	0	0	0	0
9	4	3	0	0	0	0	0	0	0
NA	14	9	1	0	0	0	0	0	0
N=1000									
3	0	0	12	0	0	0	0	0	0
4	416	422	702	1000	1000	1000	912	915	999
5	461	452	246	0	0	0	88	85	1
6	112	115	38	0	0	0	0	0	0
7	9	10	2	0	0	0	0	0	0
8	0	1	0	0	0	0	0	0	0
NA	2	0	0	0	0	0	0	0	0
N=1500									
3	0	0	2	0	0	0	0	0	0
4	450	428	787	1000	999	1000	821	846	999
5	469	505	200	0	1	0	178	154	1
6	78	65	11	0	0	0	1	0	0
7	3	2	0	0	0	0	0	0	0
N=2000									
4	385	389	831	1000	1000	1000	746	760	993
5	530	536	165	0	0	0	245	237	7
6	85	74	4	0	0	0	9	3	0
7	0	1	0	0	0	0	0	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.13

Dominant dimension frequency counts for 3-dimensional 20 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
3	57	59	53	86	88	50	58	57	40
4	238	240	515	305	309	578	272	266	520
5	418	429	357	413	413	311	422	442	377
6	287	272	75	196	190	61	248	235	63
N=1000									
3	33	33	22	42	42	25	24	31	16
4	145	167	357	255	252	460	200	189	367
5	418	409	462	423	429	397	417	405	461
6	404	391	159	280	277	118	359	375	156
N=1500									
3	18	17	8	44	41	11	22	18	6
4	144	138	303	189	190	396	144	144	333
5	385	399	475	432	434	457	436	447	486
6	453	446	214	335	335	136	398	391	175
N=2000									
3	6	6	2	32	33	15	20	17	5
4	117	114	254	166	155	338	130	136	307
5	367	377	480	414	428	491	431	432	494
6	510	503	264	388	384	156	419	415	193

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.14

Dominant dimension frequency counts for 3-dimensional 40 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	2	0	4	9	11	27	3	7	12
5	65	68	183	158	154	284	93	93	212
6	933	932	813	833	835	689	904	900	775
NA	0	0	0	0	0	0	0	0	1
N=1000									
4	0	1	2	5	5	9	1	1	3
5	24	26	55	83	81	150	33	34	82
6	976	973	943	912	914	841	965	964	914
NA	0	0	0	0	0	0	1	1	1
N=1500									
4	0	0	2	2	2	2	1	1	2
5	19	15	40	70	60	110	33	35	66
6	981	985	958	928	938	888	965	964	929
NA	0	0	0	0	0	0	1	0	3
N=2000									
4	0	0	0	0	0	1	0	0	1
5	16	12	28	52	47	85	30	26	58
6	984	988	972	948	953	914	970	974	941

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.15

Dominant dimension frequency counts for 3-dimensional 60 item datasets, simple structure.

	2			4			6		
Dominant Dimensions	*r*=0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
N=500									
4	0	0	0	1	0	1	0	0	0
5	0	1	9	13	12	34	7	5	15
6	1000	999	991	986	988	965	992	995	984
NA	0	0	0	0	0	0	1	0	1
N=1000									
5	0	0	0	2	3	9	2	2	5
6	1000	1000	1000	997	997	991	998	998	995
N=1500									
5	0	0	0	0	1	4	0	0	3
6	1000	1000	1000	1000	999	996	999	998	996
NA	0	0	0	0	0	0	1	2	1
N=2000									
5	0	0	0	1	1	4	0	0	1
6	1000	1000	1000	999	999	996	998	997	998
NA	0	0	0	0	0	0	2	3	1

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.16

Dominant dimension frequency counts for 2-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	2 RC			4 RC			6 RC		
N=2000	20	40	60	20	40	60	20	40	60
2	0	1	0	0	0	0	0	0	0
3	338	447	330	1	5	2	0	0	0
4	662	552	670	999	995	998	999	1000	1000
5	0	0	0	0	0	0	1	0	0

Note: Each column represents a given response category correlation between dimensions combination at a given level of sample size (rows).

Table D.17

Dominant dimension frequency counts for 3-dimensional datasets, mixed structure and zero correlation between dimensions

Dominant Dimensions	6 RC
N=2000	40
5	973
6	27

APPENDIX E

RANK ORDER CORRELATIONS

Table E.1

Rank Order Correlations Between Eigenvectors and True Item Locations for Simple Structure Data

	2 RC				6 RC			
N=500								
20 items	1.	2.	3.	4.	1.	2.	3.	4.
1. Eigenvector 1	1				1			
2. Eigenvector 2	0.05	1			-0.02	1		
3. True δ_{i1}	0.35	-0.86	1		-0.91	-0.27	1	
4. True δ_{i2}	-0.92	-0.32	-0.07	1	0.30	-0.91	0.00	1
N=2000								
60 items								
1. Eigenvector 1	1				1			
2. Eigenvector 2	0.03	1			-0.02	1		
3. True δ_{i1}	-0.39	-0.89	1		-0.90	0.39	1	
4. True δ_{i2}	-0.85	0.39	-0.03	1	-0.35	-0.89	-0.02	1

Table E.2

Rank Order Correlations Between Eigenvectors and True Item Locations for Complex Structure Data

	2 RC				6 RC			
N=500								
20 items	1.	2.	3.	4.	1.	2.	3.	4.
1. Eigenvector 1	1				1			
2. Eigenvector 2	-0.34	1			-0.06	1		
3. True δ_{i1}	-0.91	0.33	1		-0.75	0.39	1	
4. True δ_{i2}	0.14	0.76	-0.10	1	-0.70	-0.55	0.27	1
N=2000								
60 items								
1. Eigenvector 1	1				1			
2. Eigenvector 2	-0.24	1			0.01	1		
3. True δ_{i1}	-0.64	-0.33	1		-0.88	-0.24	1	
4. True δ_{i2}	-0.52	0.82	-0.01	1	-0.29	0.86	0.04	1

APPENDIX F

FIGURES

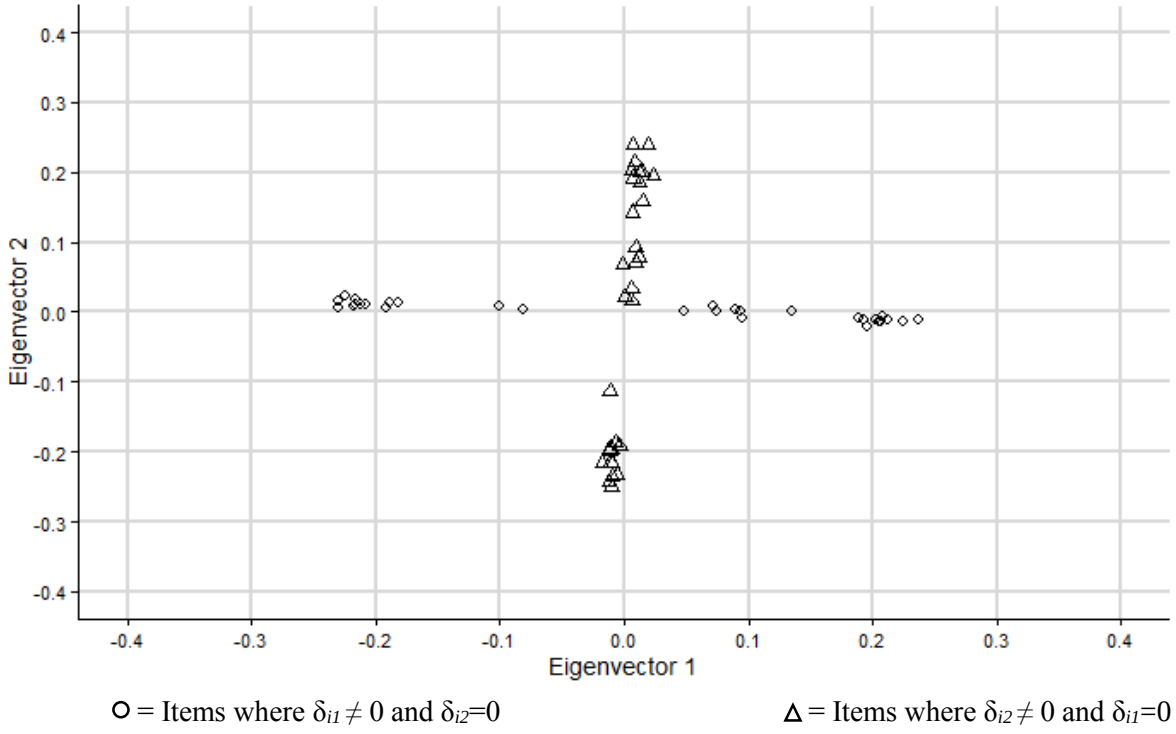


Figure F.1. Scatterplot showing the eigenvector 1 and 2 values for 60 items that correspond to dominant dimensions 1 and 2 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

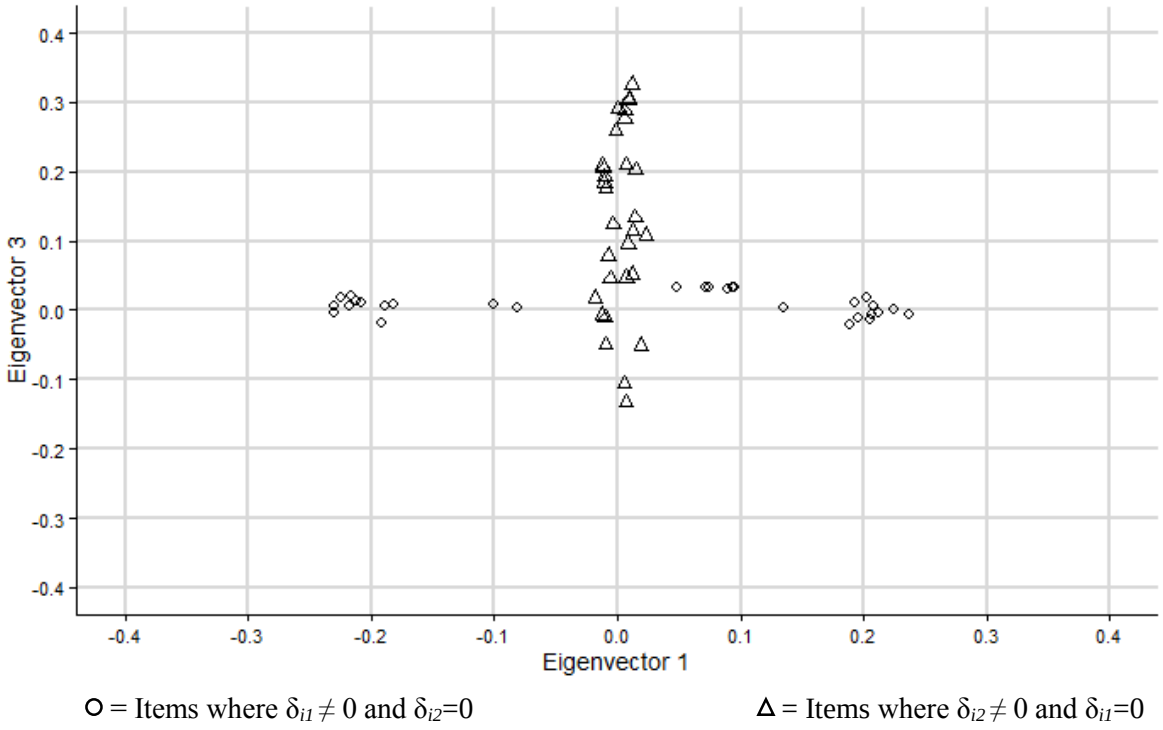
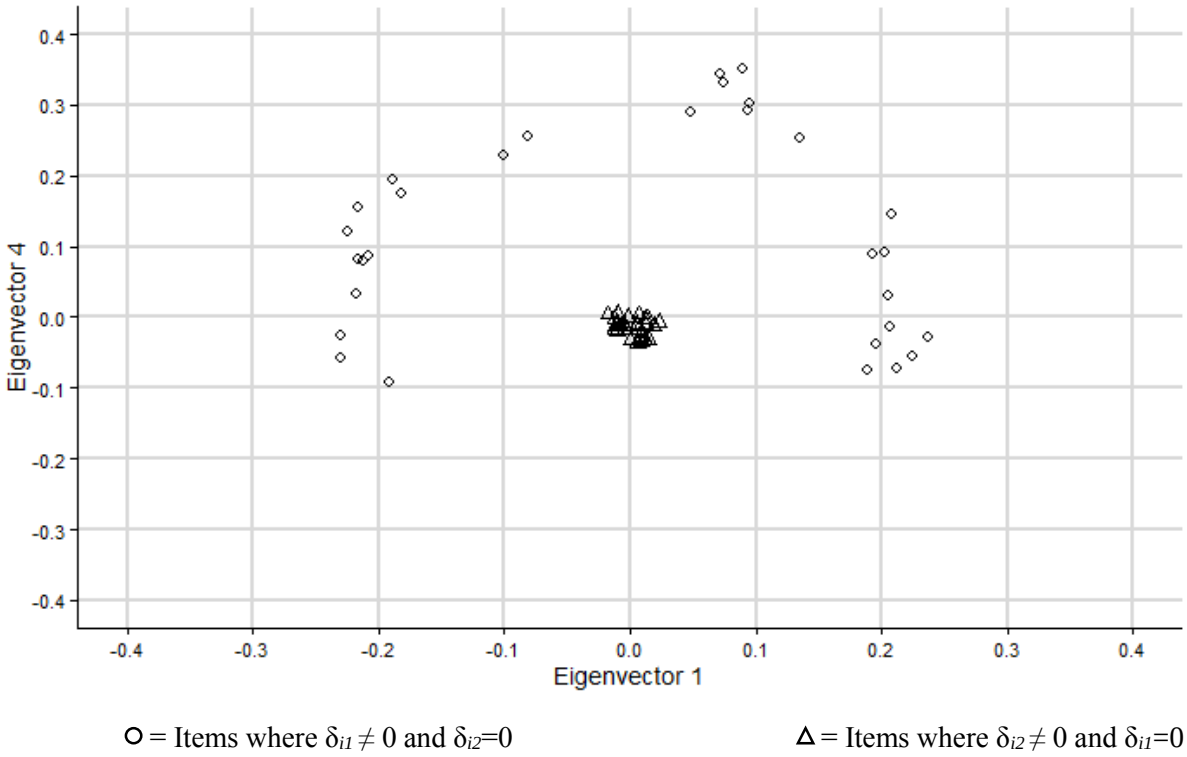
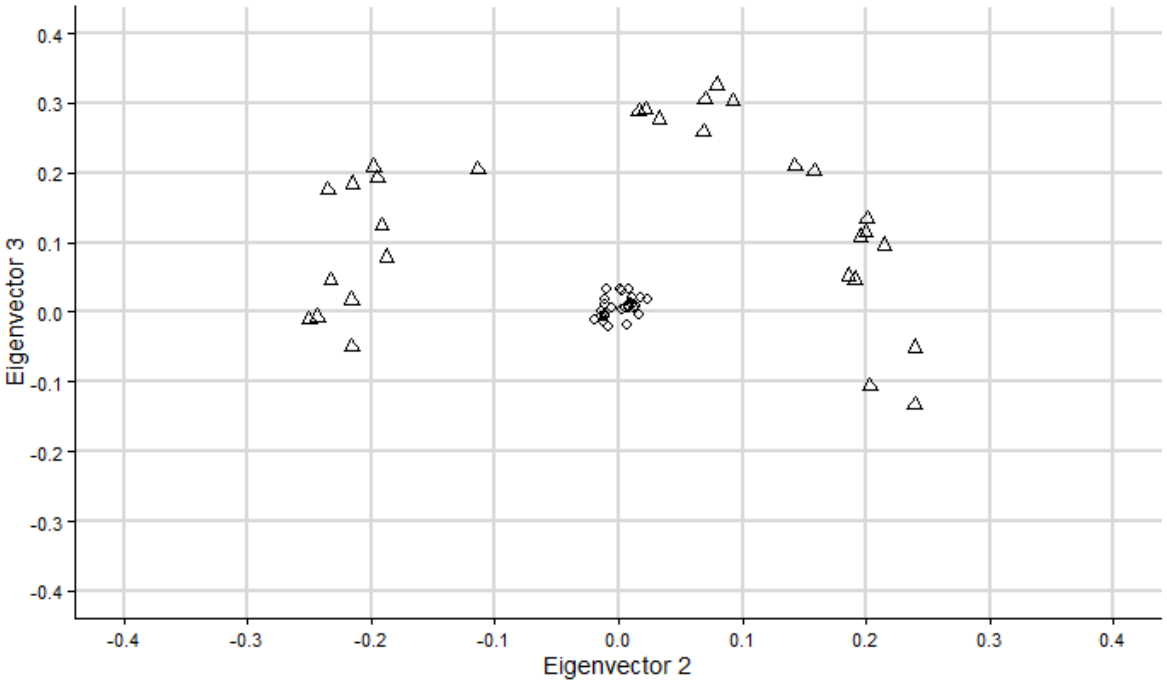


Figure F.2. Scatterplot showing the eigenvector 1 and 3 values for 60 items that correspond to dominant dimensions 1 and 3 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.



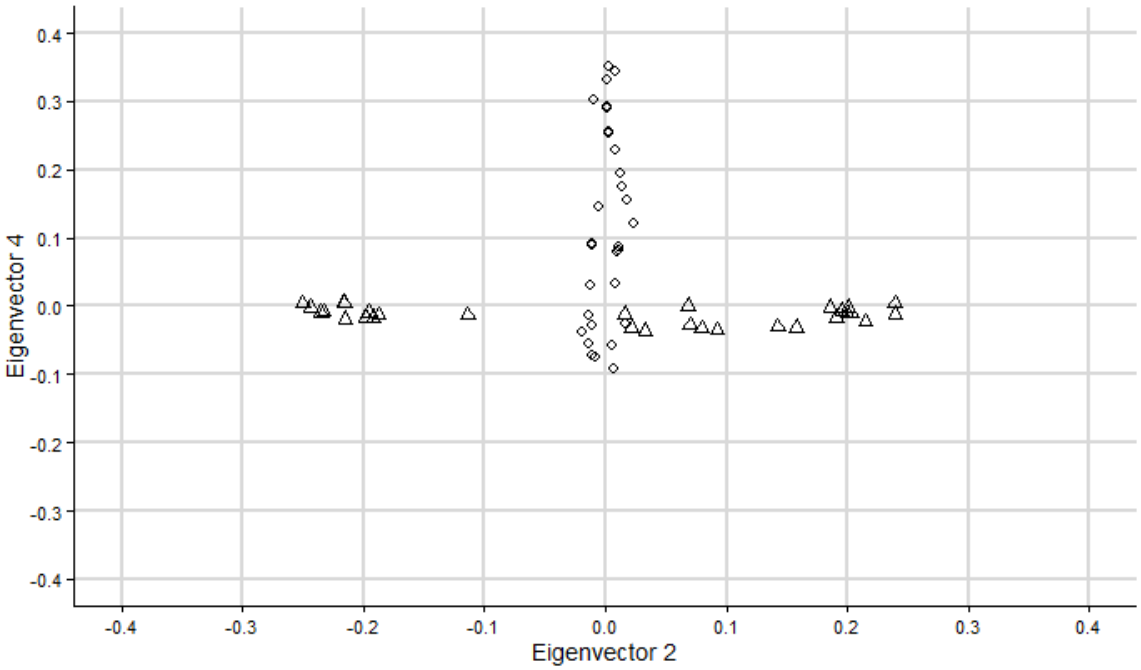
○ = Items where $\delta_{i1} \neq 0$ and $\delta_{i2}=0$ △ = Items where $\delta_{i2} \neq 0$ and $\delta_{i1}=0$

Figure F.3. Scatterplot showing the eigenvector 1 and 4 values for 60 items that correspond to dominant dimensions 1 and 4 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.



○ = Items where $\delta_{i1} \neq 0$ and $\delta_{i2}=0$ △ = Items where $\delta_{i2} \neq 0$ and $\delta_{i1}=0$

Figure F.4. Scatterplot showing the eigenvector 2 and 3 values for 60 items that correspond to dominant dimensions 2 and 3 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.



○ = Items where $\delta_{i1} \neq 0$ and $\delta_{i2}=0$ △ = Items where $\delta_{i2} \neq 0$ and $\delta_{i1}=0$

Figure F.5. Scatterplot showing the eigenvector 2 and 4 values for 60 items that correspond to dominant dimensions 2 and 4 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

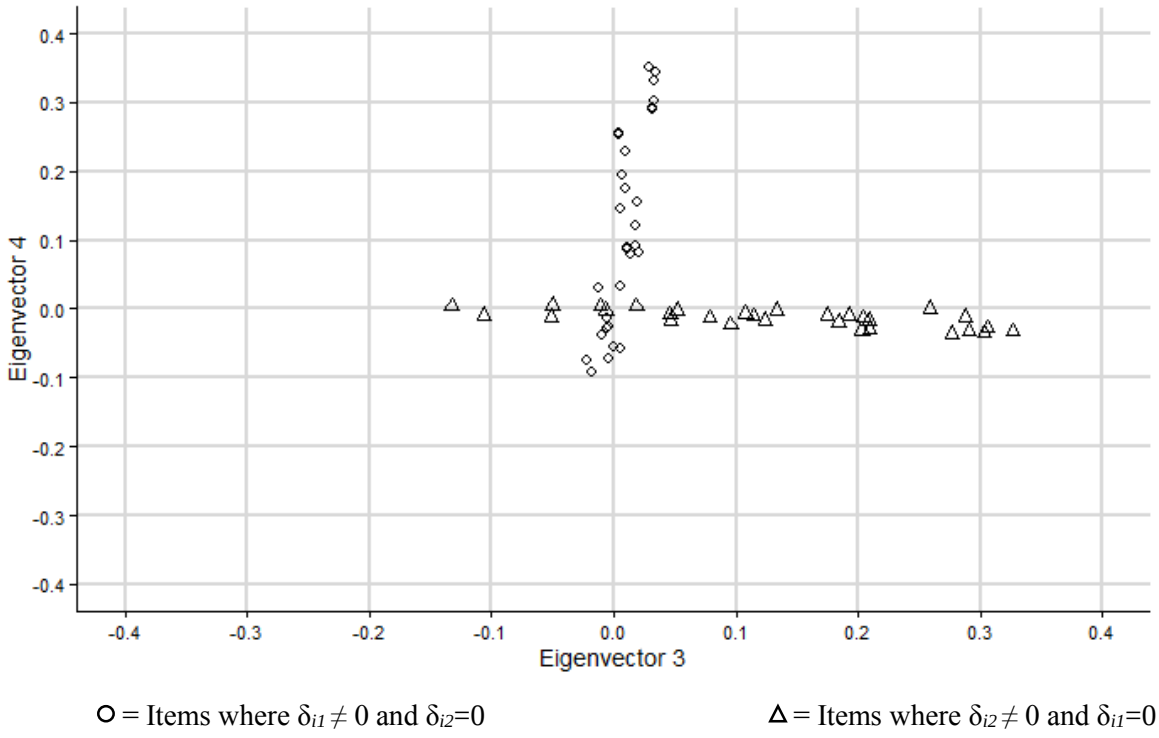


Figure F.6. Scatterplot showing the eigenvector 3 and 4 values for 60 items that correspond to dominant dimensions 3 and 4 as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, simple structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

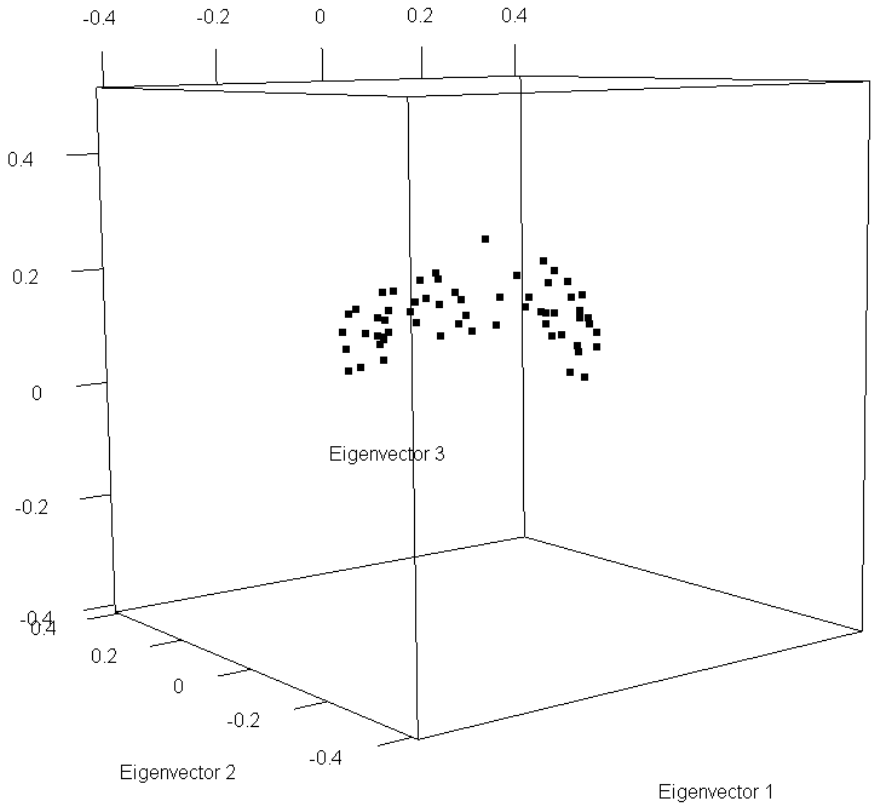


Figure F.7. Three-dimensional scatterplot showing eigenvectors that correspond to dominant dimensions for 60 items as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, complex structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

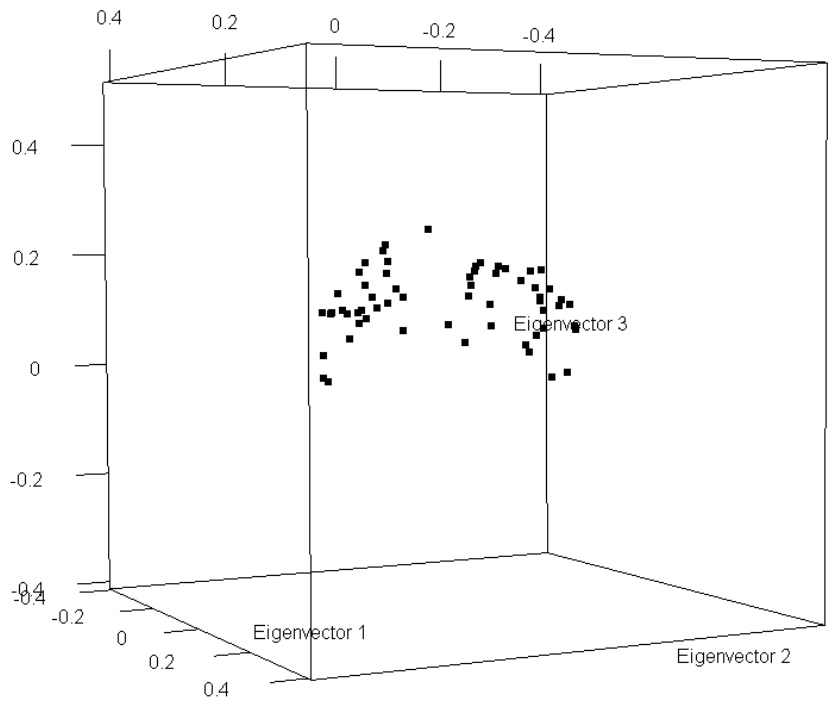


Figure F.7. Alternate view of three-dimensional scatterplot showing three eigenvectors that correspond to dominant dimensions for 60 items as identified by the PCA. Data are from a simulation study cell with 2 true dimensions, 60 items, complex structure, 2000 examinees, zero correlation between dimensions and 6 response categories.

REFERENCES

Ackerman, T. A. (1989). Unidimensional IRT calibration of compensatory and noncompensatory multidimensional items. *Applied Psychological Measurement, 13*(2), 113-127.

Andrich, D. (1988). The application of an unfolding model of the PIRT type for the measurement of attitude. *Applied Psychological Measurement,* 12, 33-51.

Andrich, D. (1996). Measurement criteria for choosing among models with graded responses. *Categorical variables in developmental research: Methods of analysis*, 3-35.

Bernstein, I. H., & Teng, G. (1989). Factoring items and factoring scales are different: Spurious evidence for multidimensionality due to item categorization. *Psychological Bulletin*, *105*(3), 467.

Buja, A., & Eyuboglu, N. (1992). Remarks on parallel analysis. *Multivariate behavioral research*, *27*(4), 509-540.

Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate behavioral research*, *1*(2), 245-276.

Chen, W. H., & Thissen, D. (1997). Local dependence indexes for item pairs using item response theory. *Journal of Educational and Behavioral Statistics*, *22*(3), 265-289.

Coombs, C. H. *A theory of data*. New York: Wiley, 1964.

Coombs, C. H., & Kao, R. C. (1960). On a connection between factor analysis and multidimensional unfolding. *Psychometrika,* 25(3), 219-231.

Davison, M. L. (1977). On a metric, unidimensional unfolding model for attitudinal and developmental data. *Psychometrika*, *42*(4), 523-548.

Froelich, A.G., & Habing, B. (2002, July). A study of methods for selecting the AT subtest in the DIMTEST procedure. Paper presented at the 2002 Annual Meeting of the Psychometrika Society, University of North Carolina at Chapel Hill.

Green, S.B. (1983). Identifiability of spurious factors using linear factor analysis with binary items. *Applied Psychological Measurement, 7(2),* 139-147.

Habing, B., Finch, H., & Roberts, J. S. (2005). A Q3 statistic for unfolding item response theory models: Assessment of unidimensionality with two factors and simple structure. *Applied Psychological Measurement, 29*(6), 457-471.

Hattie, J. (1985). Methodology review: assessing unidimensionality of tests and items. *Applied Psychological Measurement, 9*(2), 139-164.

Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika, 30*(2), 179-185.

Humphreys, L.G. (1985a) Correlations in psychological research. In D. K. Detterman (Ed.), *Current topics in human intelligence: Vol. 1. Research methodology.* 1-24. Norwood NJ: Ablex.

Kaiser, H. F. (1960). The application of electronic computers to factor analysis. *Educational and Psychological Measurement*. 20, 141-151.

Kim, H. R. (1994). New techniques for the dimensionality assessment of standardized test data (Doctoral dissertation, University of Illinois at Urbana-Champaign). *Dissertation Abstracts International, 55-12B,* 5598.

Lambert, Z. V., Wildt, A. R., & Durand, R. M. (1990). Assessing sampling variation relative to number-of-factors criteria. *Educational and Psychological Measurement, 50*(1), 33-48.

Lord, F. M. (1980). *Applications of item response theory to practical testing problems*. Hillsdale NJ: Erlbaum.

Maraun, M. D., & Rossi, N. T. (2001). The extra-factor phenomenon revisited: unidimensional unfolding as quadratic factor analysis. *Applied Psychological Measurement,* 25(1), 77-87.

McDonald, R. P. (1981). The dimensionality of tests and items. *British Journal of Mathematical and Statistical Psychology, 34*(1), 100-117.

Nandakumar, R. (1991). Traditional dimensionality versus essential dimensionality. *Journal of Educational Measurement, 28*(2), 99-117.

Nandakumar, R., & Stout, W. (1993). Refinements of Stout's procedure for assessing latent trait unidimensionality. *Journal of Educational and Behavioral Statistics, 18*(1), 41-68.

Noël, Y. (1999). Recovering unimodal latent patterns of change by unfolding analysis: Application to smoking cessation. *Psychological Methods, 4*(2), 173.

Reckase, M. (2009). *Multidimensional item response theory*. New York; London: Springer.

Reise, S. P. (1999). Personality measurement issues viewed through the eyes of IRT. In S. E. Embretson & S. L. Hershberger (Eds.), *The new rules of measurement: What every psychologist and educator should know* (pp. 219–241). Mahwah, NJ: Erlbaum.

Roberts, J. S., Donoghue, J. R., & Laughlin, J. E. (2000). A general item response theory model for unfolding unidimensional polytomous responses. *Applied Psychological Measurement*, *24*(1), 3-32.

Roberts, J. S., Lin, Y., & Laughlin, J. E. (2001). Computerized adaptive testing with the generalized graded unfolding model. *Applied Psychological Measurement*, *25*(2), 177-196.

Roberts, J. S., & Samuelson, K. (2002). *Analyzing the Structure of Binary Test Responses Using an Optimal Scaling Approach*. Paper presented at the annual meeting of the National Council on Measurement in Education, New Orleans, LA.

Roberts, J. S., & Shim, H. S. (July, 2010). *Multidimensional unfolding with item response theory: The multidimensional generalized graded unfolding model*. Paper presented at the annual meeting of the Psychometric Society, Athens, Georgia.

Ross, J., & Cliff, N. (1964). A generalization of the interpoint distance model. *Psychometrika*, *29*(2), 167-176.

Rousses, L. (1995b). *Hierarchical agglomerative clustering computer program user's manual*. Urbana-Champaign: Statistical Laboratory for Educational and Psychological Measurement, Department of Statistics, University of Illinois.

Stark, S., Chernyshenko, O. S., Drasgow, F., & Williams, B. A. (2006). Examining assumptions about item responding in personality assessment: Should ideal point methods be considered for scale development and scoring? *Journal of Applied Psychology*, *91*(1), 25.

Stout, W. (1987). A nonparametric approach for assessing latent trait unidimensionality. *Psychometrika*, *52*(4), 589-617.

Stout, W. (2002). Psychometrics: From practice to theory and back. Psychometrika, 67(4), 485-518.

Stout, W., Froelich, A. G., & Gao, F. (2001). Using resampling methods to produce an improved DIMTEST procedure. In *Essays on item response theory*, 357-375. Springer New York.

Stout, W., Habing, B., Douglas, J., Kim, H. R., Roussos, L., & Zhang, J. (1996). Conditional covariance-based nonparametric multidimensionality assessment. *Applied Psychological Measurement*, *20*(4), 331-354.

Tate, R. (2003). A comparison of selected empirical methods for assessing the structure of responses to test items. *Applied Psychological Measurement*, *27*(3), 159-203.

Tay, L., & Drasgow, F. (2012). Theoretical, statistical, and substantive issues in the assessment of construct dimensionality: Accounting for the item response process. *Organizational Research Methods*, *15*(3), 363-384.

Tay, L., Drasgow, F., Rounds, J., & Williams, B. A. (2009). Fitting measurement models to vocational interest data: Are dominance models ideal? *Journal of Applied Psychology*, *94*(5), 1287.

Thompson, V. M. (2014). Marginal Bayesian estimation in the multidimensional generalized graded unfolding model. (Unpublished doctoral dissertation). Georgia Institute of Technology, Atlanta, GA.

Thurstone, L. L. (1928). Attitudes can be measured. *American Journal of sociology*, 529-554.

Van Abswoude, A. A., van der Ark, L. A., & Sijtsma, K. (2004). A comparative study of test data dimensionality assessment procedures under nonparametric IRT models. *Applied Psychological Measurement*, *28*(1), 3-24.

Van Schuur, W. H., & Kiers, H. A. (1994). Why factor analysis often is the incorrect model for analyzing bipolar concepts, and what model to use instead. *Applied Psychological Measurement*, 97-110.

Waller, N. G., Tellegen, A., McDonald, R. P., & Lykken, D. T. (1996). Exploring nonlinear models in personality assessment: Development and preliminary validation of a negative emotionality scale. *Journal of Personality*, 64, 545-576.

Yen, W. M. (1985). Increasing item complexity: A possible cause of scale shrinkage for unidimensional item response theory. *Psychometrika*, *50*(4), 399-410.

Zhang, J., & Stout, W. (1999). The theoretical DETECT index of dimensionality and its application to approximate simple structure. *Psychometrika*, *64*(2), 213-249.